0.2 | Set and Interval Notation

Sets: A set is a well-defined collection of objects we call *elements*. Well-defined means that it is possible to determine if something belongs to the set or not.

How to describe a set: There are three main ways to describe a set.

- **The Verbal Method**: Use a sentence to describe a set
  "*The integers one through ten.*"
- **The Roster Method**: List each element between curly braces and separate using commas. Order does not matter with sets, and repetition is not allowed.
  $\{6, 1, 9, 2, 3, 5, 7, 8, 10, 4\}$
- **The Set-Builder Method**: Within curly braces, provide a variable(s), and then give constraints on said variable(s).
  $\{x \mid 1 \leq x \leq 10, x \text{ is an integer}\}$

Common Symbols:

- $\in$
  - If x is an element of a set A we write $x \in A$ and read 'x is in A.'
- $\notin$
  - If x is not an element of a set A we write $x \notin A$ and read 'x is not in A.'
- $\subseteq$
  - Given two sets A and B, A is a **subset** of B written '$A \subseteq B$' if every element of A is also an element of B.
- $\cap$
  - The **intersection** of A and B is written $A \cap B$ and describes every element found in *both A and B*. More formally we can write $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- $\cup$
  - The **union** of A and B is written $A \cup B$ and describes every element found in *either A or B or both*. More formally we can write $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- $\emptyset$
  - The **empty set** is the set which contains no elements. More formally $\emptyset = \{x \mid x \neq x\}$.

Real Numbers: A real number is any number which can be written as a decimal. The set of real numbers is denoted by $\mathbb{R}$ and is sometimes read as "all real numbers."

Special Subsets of Real Numbers: $\mathbb{R}$ is really the only set you need to be familiar with for this course, but I have provided a list of other common subsets for those curious.

- **The Natural Numbers**[1]: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ The ellipsis '. . . ' simply implies that the numbers go on forever.
- **The Integers**: $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- **The Rational Numbers**: $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z} \text{ and } b \in \mathbb{Z} \text{ where } b \neq 0\}$, put more simply this is the set of all fractions.
- **The Irrational Numbers**: $\mathbb{P} = \{x \mid x \in \mathbb{R} \text{ but } x \notin \mathbb{Q}\}$. This set will contain numbers like π and $\sqrt{2}$.

Interval Notation: Interval notation is a standardized way to describe intervals on the real number line. In any interval you will see two numbers, the first is the lower bound, or the smallest number that can be in the interval, and the second is the upper bound, or the largest umber that can be in the interval.

- **Example**: The following interval: $[1, 10]$ describes the real numbers 1 through 10. Keep in mind, that these are real numbers, not just integers, so the number 3.14159 is valid and falls within the interval.

Parenthesis vs Brackets: When using interval notation, we close off the interval using parenthesis or brackets depending on if we want to include the bounds of the interval. A bracket implies that the number at the very end of the interval *is* included, while a parenthesis implies that the number at the very end of the interval *is not* included. Every number right before it will be included, but the number itself will not be.

- **Example**: Lets change the interval from the previous example to have one parenthesis. The interval will now be $[1, 10)$. In this interval, 1 is included, but 10 is not. The number 9.999999999999999999 however, will be included.

Combining Intervals: If we want to combine two intervals, we can use the $\cup$ symbol. This is useful for joining intervals but skipping any thing in between them.

- **Example**: Say we want bring to fruition the saying "*one two skip a few, ninety-nine one hundred.*" In order to do this we will need to join two intervals together, the first being $[1, 2]$ and the second being $[99, 100]$. We can join the intervals using the union symbol like so: $[1, 2] \cup [99, 100]$. Now, any number between 1 and 2 will be in our interval, and so will any number between 99 and 100. Any number after 2 and before 99, however, will not be included. So numbers like 50, 2.0001 or π, will not be in our interval.

[1]The official position of your book is that the natural numbers *do not* contain 0. I however cannot stand for this and will be putting 0 as a natural number on this worksheet. There is an ongoing debate among mathematicians about the status of 0 in the natural numbers, but I can tell you the absolute truth that 0 is a natural number and anyone who thinks otherwise is wrong.