0.4 | Factoring Quadratics

Why Factor? Before we actually go ahead and review factoring, let's take a moment to remind ourselves why factoring is such a powerful technique. When we factor a quadratic (or any polynomial), we can take advantage of something known as the **zero product property**. The zero product property is simple, but powerful. It states that if $a \cdot b = 0$, then either $a = 0$ or $b = 0$ (or both). What this means, is that if we are able to turn a quadratic function into a *multiplication of factors*, we can *solve each factor individually* and find solutions by solving much simpler equations.

For example: the quadratic $x^2 - 6x - 91 = 0$ might look really hard to solve directly (and it is), but if we can factor it, we find that it is equal to $(x+7)(x-13) = 0$. We can then solve individually $(x+7) = 0$ and $(x-13) = 0$, to find that $x = -7$ and $x = 13$.

Basic Factoring Technique: The most basic, and most fundamental factoring technique, involves solving a miniature number puzzle. Let's warm up with a few examples:

- Find two numbers that multiply to equal 9, but that add to equal 20.
- Find two numbers that multiply to equal -2, but that add to equal -8.

This simple puzzle is a reverse engineered version of factoring. Think about what happens if we expanded the terms $(x+4)(x+5)$. We start by multiplying the two x's to get x^2, then we would need to add together two terms $4x$ and $5x$ to equal $9x$, finally we will need to multiply together 4 and 5 to get 20. The complete answer is $x^2 + 9x + 20$. Hopefully the idea is becoming clear.

To find the factors of a quadratic $x^2 + bx + c$, find two numbers which add together to get b, and that multiply together to get c.

Once you find these two numbers, let say they are n and m, simply write out the factors as individual terms inside parenthesis: $(x+n)(x+m)$.

1. Factor the quadratic: $x^2 - 2x - 8$ *(Hint: you should already have the values earlier on this page!)*

2. Factor the quadratic: $z^2 - 10z + 21$

3. Factor the quadratic: $y^2 + 16y + 60$

4. Factor the quadratic: $x^2 + 14x + 49$

This method however, has one major weakness. Notice that all the examples so far have no coefficient next to the x^2 term, so what do we do in the case which $a \neq 1$?

Next Page ↓

The AC Method: The AC method[1] is a relatively simple way to adjust for a quadratic having the a coefficient be larger than 1. Lets briefly scope out the steps and then look at an example.

For a quadratic of the form $ax^2 + bx + c$

- **Step 1**: Multiply a and c, and rewrite the quadratic as: $x^2 + bx + ac$.
- **Step 2**: Factor this new quadratic.
- **Step 3**: Within the factors, divide the constant terms by a.
- **Step 4**: If desired, rewrite the factors with integer coefficients

Lets follow along with an example:
Factor the quadratic using the AC method

$$6x^2 + 11x + 3$$

Step 1: We start by multiplying 6 by 3, this product equals 18. We will then remove the coefficient 6 from x^2, and replace 3 with our new product 18.

$$x^2 + 11x + 18$$

Step 2: Now we can simply factor this new quadratic. We need to find two numbers which add to 11, but multiply to 18. These values will be 9 and 2. We can then write the factors out like so:

$$(x + 9)(x + 2)$$

Step 3: We now divide the constant terms of these factors by the original a value. In our case $a = 6$, so we divide the +9 and +2 by 6.

$$\left(x + \tfrac{9}{6}\right)\left(x + \tfrac{2}{6}\right)$$

It is then often useful to reduce the fractions if we are able

$$\left(x + \tfrac{3}{2}\right)\left(x + \tfrac{1}{3}\right)$$

Step 4 (optional): The last step is technically optional, but is almost always helpful. If the division we do in step 3 doesn't reduce to a integer, we can rewrite the factor with integer coefficients. Set the factor equal to zero, then multiply by whatever integer you would like (ideally the integer which is the denominator of the fraction you are trying to reduce). You do not need to use the same integer for both factors.

$$2\left[x + \tfrac{3}{2}\right] = 2[0] \text{ and } 3\left[x + \tfrac{1}{3}\right] = 3[0]$$

Distribute the value and write as factors. This completes the AC method.

$$(2x + 3)(3x + 1)$$

Due to the more complicated solutions when using the AC method, we can always check our work by expanding the solution and verifying that it equals the original quadratic.

[1] There are more than one versions of the AC method, and if you prefer to use a different one over this one that is totally fine. This version of the method is sometimes called "*slide and divide.*"

5. Use the AC method to factor: $3x^2 + 19x + 20$

6. Use the AC method to factor: $5x^2 + 14x - 3$

7. Use the AC method to factor: $6t^2 - 19t - 7$

8. Use the AC method to factor: $4z^2 + 19z + 12$

Further Notes: Factoring is powerful, but no matter what *human usable*[2] factoring method you use, it will have some unavoidable shortcomings. For one, larger values quickly become much more difficult to identify the "number puzzle" solution. One option is to list out a factoring tree for each value, but after a certain point this will be much too time consuming. Another shortcoming, is that (human) factoring methods really only work for integers, or very nice fractions. Decimals, or irrational numbers, can increase the difficulty even further. Never feel bad about resorting to the quadratic formula if you believe factoring is too difficult. Even deceptively "nice" quadratics can be impossible to factor.

9. Explain why $x^2 + 2x + 2 = 0$ cannot be factored *(Hint: Use the discriminant).*

[2]Computers are much better at factoring than we are, however even computers can have trouble! The difficulty of factoring large numbers is the basis for how things like your credit card information is secured.