1.1 | Functions and their Representations

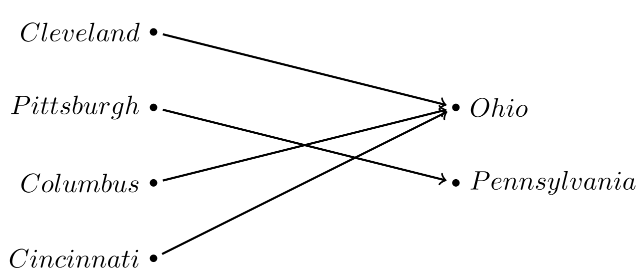
Brief[1] Set Notation Review: A *set* is a collection of arbitrary objects called *elements*. There are three main ways to describe a set.

- **The Verbal Method**: Describe the set using words.
  - *"The natural numbers one through ten."*
- **The Roster Method** List the elements between curly braces.
  - $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- **The Set-Builder Method**: A two step description within curly braces which includes a variable and then conditions on said variable.
  - $\{x \mid 1 \leq x \leq 10,\ x \text{ is an integer}\}$

Functions: A *function* is a process by which elements of one set are assigned ("mapped") to one and only one element of another set. The wording "one and only one" is important here. If the function maps an element to zero, two, or more elements then it fails to be a function.

Domain: The *domain* is the set of inputs. The domain variable you are probably most used to is x, but you can use anything you would like.
Range: The *range* is the set of outputs. If x is an element of the domain, then for a function f, $f(x)$ will describe that elements spot in the range.

Mappings: One of the more basic ways to represent a function is with something called a *mapping diagram*. This is when the domain and range are written down and arrows are used to describe the mappings.



Notice in the example above that each domain element is mapped to one and only one range element. This means the mapping is a valid function. Think about what would happen if we reversed the domain and range. If we mapped states to their cities, then we would have the element *Ohio* mapping to three different elements! This would not pass as a valid function.

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[1] This review is *very brief*. For a more complete review of set notation see section A.1 of the textbook on page 727.

1. Come up with your own mapping diagram and draw it below. Once finished, ask yourself: is it a valid function? If it is not, how would you need to change it in order for it to satisfy the definition of a function?

Minimum: The *minimum* of a function is the smallest possible output. More concretely, m is a minimum if for every value of x in the domain $m \leq f(x)$.
Maximum: The *maximum* of a function is the largest possible output. More concretely, M is a maximum if for every value of x in the domain $f(x) \leq M$.

Algebraic Representations of Functions: Functions can also be represented algebraically. This is what you are most likely used to. An equation of one variable in terms of another, $y = x$ is one example. If we are given an algebraic equation and asked to verify if it is a function, we will want to isolate the range variable (traditionally y) and ensure that we will only get one associated x-value back.

2. Verify if the equation is a function: $x^3 + y^2 = 25$

3. Verify if the equation is a function: $y - 1 = x^2$

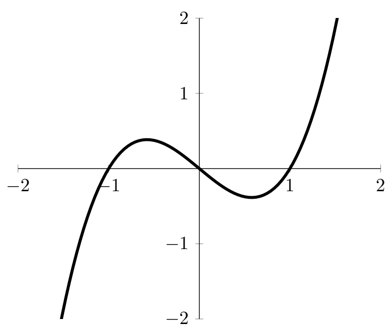
4. Verify if the equation is a function: $y^2 - 1 = x$

Geometric Representations of Functions: Finally, we can look at the graph of a function to see if it is valid.

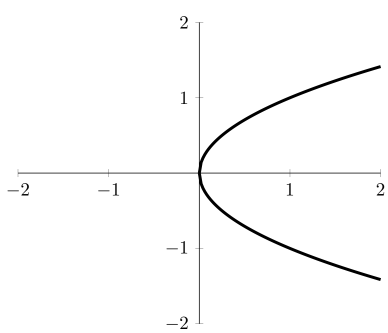
Textbook Theorem 1.1. The Vertical Line Test:
A graph in the xy-plane represents y as a function of x if and only if no vertical line intersects the graph more than once.

In English 1.1. The Vertical Line Test:
Place your pencil vertically on the graph. Move it from left to right hitting every part of the graph possible. The graph should only ever be touching your pencil at one point. If it touches your pencil two or more places then it is not a function.

5. Use the vertical line test to determine if the graph is a function.



6. Use the vertical line test to determine if the graph is a function.



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