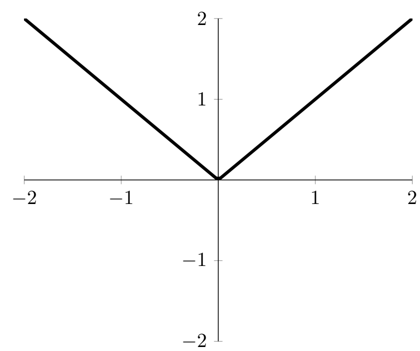
1.3 | Absolute Value Functions

Piecewise Functions Review: A *piecewise* function is defined using different equations on multiple intervals. To graph a piecewise function, simply treat the equations and intervals as a set of instructions, graphing each line only where it is defined by the interval.

Absolute Value Functions: The absolute value of a number x, written $|x|$, is defined by the following equation:

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

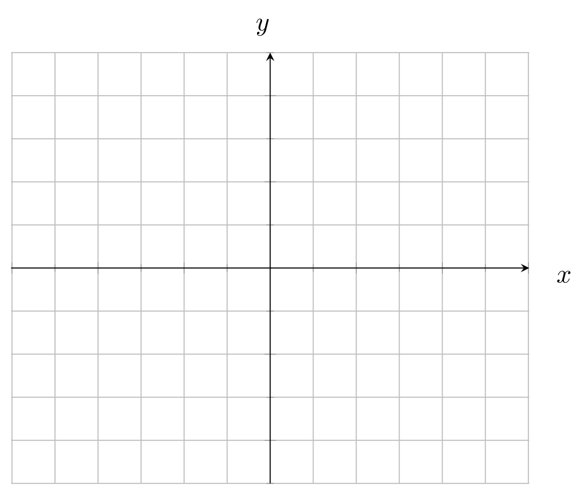
With more complicated absolute value equations, x may become more than a number, it can become an equation. When expanding the definition of an absolute value function into piecewise form, make sure to replace "x" with every instance of the contents of your absolute value bars.



$f(x) = x$

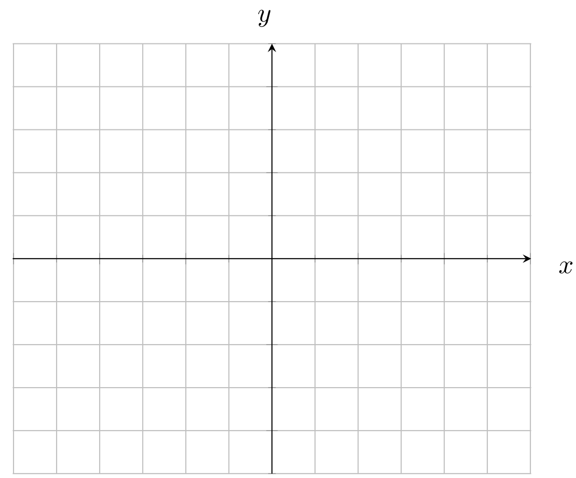
> **Problem Solving Tip 1. Piecewise form**
> Absolute value problems give lots of students trouble because they show up so early in the course, but will return in later sections. If you ever find yourself stuck on an absolute value problem, being able to convert to piecewise form is almost guaranteed to help you (if not be apart of the solution itself). This formula is one of the things in this course that is useful to have memorized for every test.

1. **Worked Example**: Graph the function: $f(x) = |x + 4|$

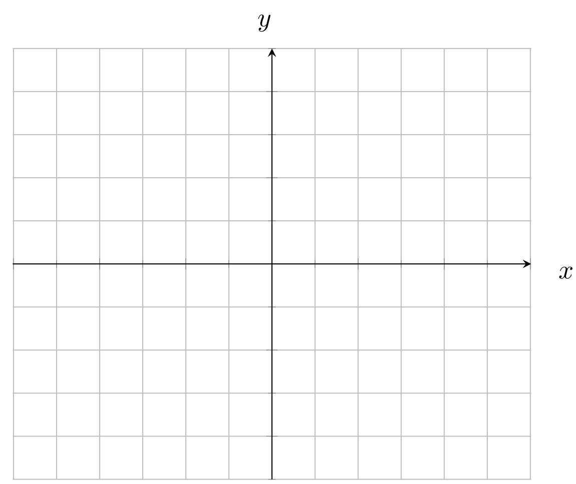


Scan the QR code for a video solution

2. Graph the function: $f(x) = |x| + 4$



3. Graph the function: $g(t) = 3|t + 4| - 4$



Solving equations with absolute values: When solving an equation that involves an absolute value, convert the side of the equation containing the absolute value into its piecewise definition, then solve both versions of the equation using the two definitions in the piecewise function. If both sides of the equation involve an absolute value, you may find use of this important property:

$$\text{if } |a| = |b| \text{ then } a = \pm b$$

4. Solve the equation: $|4 - x| = 7$

5. Solve the equation: $4 - |t| = 3$

6. Solve the equation: $|7t - 1| + 2 = 0$

7. Solve the equation: $|x - 4| = x - 5$

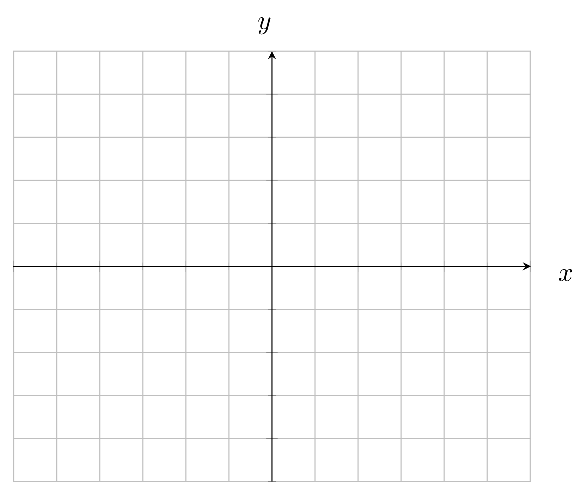
8. Solve the equation: $|3x - 2| = |2x + 7|$

9. Solve the equation: $|1 - 2x| = |x + 1|$

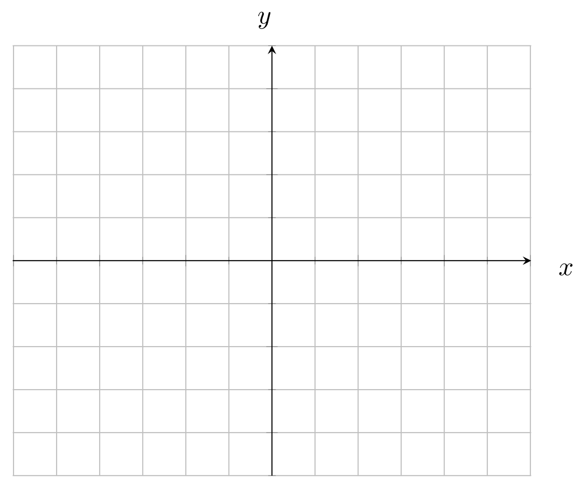
Graphical Interpretations of Equations and Inequalities: By looking at a graph we can identify areas where functions are equal to, greater than, or less than other functions. Assume that we are comparing two functions, $f(x)$ and $g(x)$. Then on the graph the following situations hold:

- Any intersection of the two graphs implies $f(x) = g(x)$
- If f is *below* g on the graph it implies $f(x) < g(x)$ for that interval.
- If f is *above* g on the graph it implies $f(x) > g(x)$ for that interval.

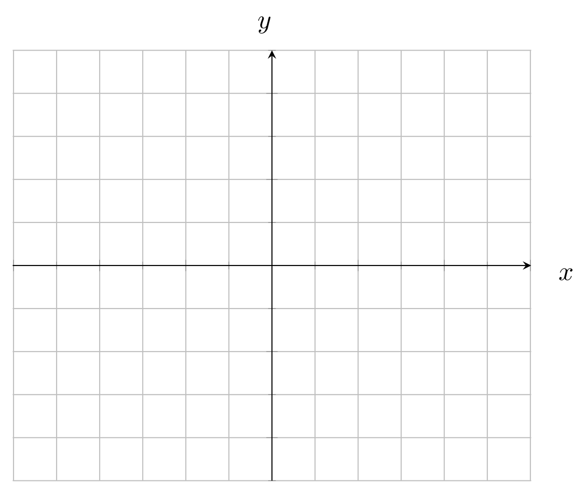
10. Solve the inequality by graphing: $|3x - 5| < 4$



11. Solve the inequality by graphing: $|2 - t| - 4 \geq -3$



Challenge Problem: Graph the function: $f(x) = |x + 2| - |x + 4|$



Solution

Materials in SI are not a suitable replacement for materials in class. These materials are not for use on exams.