MATH1300
Selected Challenge Problems
Volume I

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Preface:

These problems are a compilation of problems from the textbook, along with some of my own creations, designed to form multi-step problems that provide a decent challenge to anyone in MATH1300. My goal is that if one is able to complete more than one problem in each section, they should be adequately prepared for the exam. *Generally* speaking, the problems become more difficult as you move from **A** to **F**, although some students may find earlier problems more difficult and later problems easier. If you find yourself struggling to start *any* problem at all, you may want to go back and review easier questions from the book or my other worksheets before returning to these problems. Problems with an **asterisk*** (usually problem **F**) are exceptionally difficult and require a deeper level of introspection into the topic to solve. If you are able to solve a problem with an asterisk, you likely have enough knowledge of the given section to perform well on the exam (no promises).

Keep in mind, I have no special insider information on what will actually appear on the exam, and **you should not take this booklet as a representation of what you will see on your exam.**

Roman

1.1

A Observe the following equation: $2xy = 4$.

i. Does this equation represent y as a function of x?

ii. If so, write the domain of the equation as set, if not, provide an example where it fails as a function.

D Observe the set of ordered pairs

$$\{(-3,9),(1,1),(3,1),(0,0),(-2,4),(-3,7),(4,0)\}$$

i. Does the set of ordered pairs represent a function?

ii. If so, write the domain as a set, if not, provide an example where it fails as a function.

C Observe the following data table.

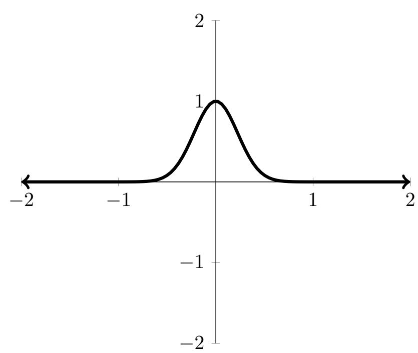
x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

i. Does the given table represent y as a function of x? Explain.

ii. Write the domain of the table as a set.

iii. Write the range of the table as a set.

D Observe the graph

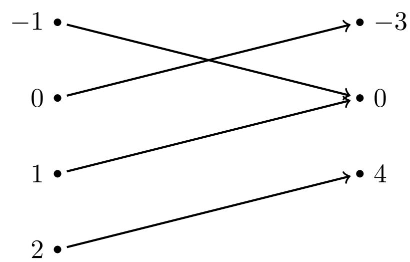


i. Does the graph represent a function? Explain.

ii. Write the domain of the graph using interval notation.

iii. Write the range of the graph using interval notation.

E Consider the function f as a mapping diagram shown:



i. Write the domain of f as a set.

ii. Write the range of f as a set.

iii. Find $f(0)$ and solve $f(x) = 0$.

iv. Write f as a set of ordered pairs.

F Let $g = \{(-1, 4), (0, 2), (2, 3), (3, 4)\}$

i. Write the domain of g as a set.

ii. Write the range of g as a set.

iii. Find $g(0)$ and solve $g(x) = 0$.

iv. Create a mapping diagram for g.

1.2

A Graph the function $h(t) = \frac{2}{3}t + \frac{1}{3}$.

i. What is the slope?

ii. State the axis intercepts, if they exist.

B Graph the function $j(w) = \dfrac{1-w}{2}$

i. What is the slope?

ii. State the axis intercepts, if they exist.

C Find the equation of the function that contains the points $(1,3)$ and $(3,2)$.

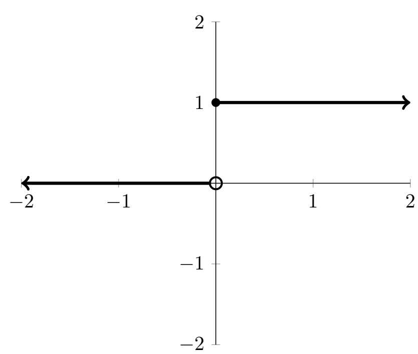
D Graph the piecewise function $f(x) = \begin{cases} 4-x & \text{if } x \leq 3 \\ 2 & \text{if } x > 3 \end{cases}$

i. Write the domain in interval notation.

ii. Write the range in interval notation.

iii. State the axis intercepts, if they exist.

E The unit step function is graphed below:



i. Write the equation $U(t)$ of the unit step function.

ii. Write the domain of $U(t)$

iii. Write the range of $U(t)$

F* Explain why the graph of a function $f(x)$ must have at most one y-intercept.

1.3

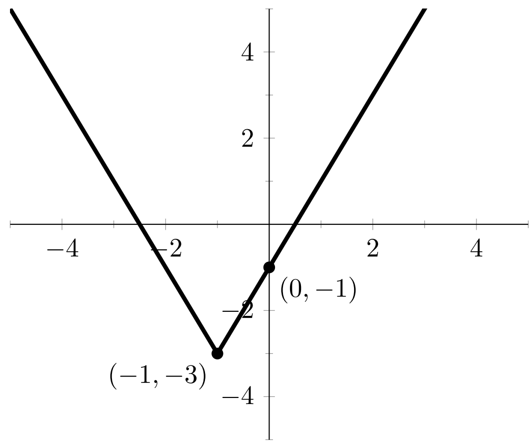
A Graph the function $g(t) = 3|t+4| - 4$

i. Write the domain of $g(t)$ in interval notation.

ii. Write the range of $g(t)$ in interval notation.

iii. State the axis intercepts, if they exist.

B The graph of $F(x)$ is shown below:



i. Write piecewise function definition of $F(x)$.

ii. State the domain of $F(x)$.

iii. State the range of $F(x)$.

C Graph the function $g(x) = |t+4| + |t-2|$.

i. Write the domain of $g(x)$ using interval notation.

ii. Write the range of $g(x)$ using interval notation.

iii. State axis intercepts, if they exist.

D Solve the equation $|3x - 2| = |2x + 7|$.

i. Write the solutions as a set.

E Given $f(x) = |3x - 5|$ and $g(x) = 4$

i. Graph $f(x)$.

ii. Graph $g(x)$ (on the same plot).

iii. Solve $f(x) \leq g(x)$. Write your answer in interval notation.

F* Show that if d is a real number with $d > 0$, the solution to $|x - a| < d$ is the interval: $(a - d, a + d)$. That is, an interval centered at a with 'radius' d.

1.4

A Let $f(x) = x^2 - 2x - 8$

i. Complete the square on $f(x)$.

ii. Write the vertex.

iii. Find the axis intercepts.

iv. Graph $f(x)$.

B Let $h(t) = -3t^2 + 5t + 4$

i. Compute the discriminant of $h(t)$. How many real zeros does $h(t)$ have?

ii. Find the zero(s) of $h(t)$ if they exist, write your solutions as a set.

C Let $g(x) = x^2 - 3x + 9$

i. Is $g(x)$ factorable?

ii. If yes, write $g(x)$ in factored form. If not, explain why.

D Solve the inequality $3x^2 \leq 11x + 4$, write your answer in interval notation.

E Solve the inequality $5t + 4 \leq 3t^3$, write your answer in interval notation.

F* Graph $f(x) = |1 - x^2|$.

2.1

A Let $g(x) = 3x^5 - 2x^2 + x + 1$

i. Identify the degree of $g(x)$.
ii. Identify the leading coefficient of $g(x)$.
iii. Identify the leading term of $g(x)$.
iv. Identify the constant term of $g(x)$.
v. Write the end behavior of $g(x)$.

B Let $f(x) = 3(x + 2)^3 + 1$

i. Write the parent function $P(x)$ for $f(x)$.
ii. Pick three points from the parent function $P(x)$ and apply the transformations of $f(x)$ to write three points on the graph of $f(x)$.
iii. Sketch the graph of $f(x)$.
iv. State the domain and range of $f(x)$ using interval notation.

C Let $f(x) = 2 - 3(x - 1)^4$

i. Write the parent function $P(x)$ for $f(x)$.
ii. Pick three points from the parent function $P(x)$ and apply the transformations of $f(x)$ to write three points on the graph of $f(x)$.
iii. Sketch the graph of $f(x)$.
iv. State the domain and range of $f(x)$ using interval notation.

D Let $h(t) = t^2(t - 2)^2(t + 2)^2$

i. List all zeros of $h(t)$ and their corresponding multiplicities.
ii. Write the end behavior of $h(t)$.
iii. Sketch a graph of the function $h(t)$.

E Let $g(x) = (2x + 1)^2(x - 3)$

i. List all zeros of $g(x)$ and their corresponding multiplicities.
ii. Write the end behavior of $g(x)$.
iii. Sketch a graph of the function $g(x)$.

F Let $f(x) = (x^2 + 1)(x - 1)$

i. Determine analytically if $f(x)$ is even, odd, or neither.

2.2

A Let $f(z) = 4z^3 + 2z - 3$ and $g(z) = z - 3$

i. Compute $f(z)/g(z)$.

ii. Write $f(z)$ as an expression involving $g(z)$, a quotient, and remainder (if it exists).

B Let $f(x) = 2x^3 - x + 1$ and $g(x) = x^2 + x + 1$

i. Compute $f(x)/g(x)$.

ii. Write $f(x)$ as an expression involving $g(x)$, a quotient, and remainder (if it exists).

C Let $a(x) = x^4 - 6x^2 + 9$ and $b(x) = (x - \sqrt{3})$

i. Compute $a(x)/b(x)$.

ii. Write $a(x)$ as an expression involving $b(x)$, a quotient, and remainder (if it exists).

D Let $g(z) = z^3 + 2z^2 - 3z - 6$ be a polynomial function with a known real zero of $c = -2$

i. Find the remaining real zeros of $g(z)$

E Let $x^3 - 6x^2 + 11x - 6$ be a polynomial function with a known real zero of $c = 1$

i. Find the remaining real zeros of $g(z)$

F* Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with the property that $a_n + a_{n-1} + \cdots + a_1 + a_0 = 0$. (That is, the sum of the coefficents and the constant term is 0.)

i. Show that $(x - 1)$ is a factor of $f(x)$.