5.1 | Graphs of Functions

Throughout the sections so far, we have seen many different types of functions and the methods we can use to analyze each one. In this section, we introduce a variety of definitions that we can use to describe *any* function. Some terms will be review, but will still be included so all vocabulary terms are in one common place.

Domain: The domain of a function is the set of *valid* inputs. If an input value leads to any sort of issue, it is not included in the domain (for example, dividing by zero, or a square root of a negative). Domain values are found on the horizontal (x) axis.

Range: The range of a function is the set of outputs that result from operating on the inputs. Range values are found on the vertical (y) axis.

Absolute Maximum: The absolute maximum of a function is the largest output from a function (if there is one).

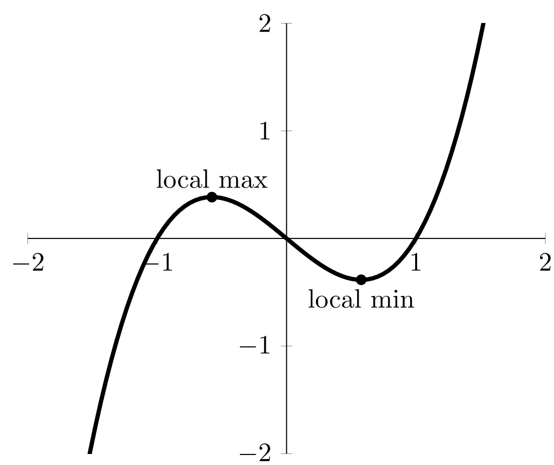
Absolute Minimum: The absolute minimum of a function is the smallest output from a function (if there is one).

Local Maximum: A point (a, b) is a local maximum if you can define an *open interval* on the domain which contains a, such that b is the maximum function value on that interval.

Local Minimum: A point (a, b) is a local minimum if you can define an *open interval* on the domain which contains a, such that b is the minimum function value on that interval.

More on meaning of "local": Graphically, the local maximum/minimum will be the various peaks and valleys throughout a function.

We can make this definition rigorous from a mathematical standpoint by requiring that you can define an *open interval* on the domain where the function value is a max/min. The fact that we use an open interval here is important. An open interval is one which does not include its endpoints, and if we do not include endpoints, we ensure that the end points will not be potential maximums or minimums.[1]



[1] To understand why this works. Think about the interval $(0, 1)$ and ask: "What is the largest number in this interval?" You cannot pick 1, as it is not included. You can try picking 0.999, but you can always add more 9's (say, 0.9999), and you will never be able to pin down a specific value which we can call a maximum.

Increasing: A function is increasing over an interval if as the inputs increase (x gets larger) the outputs increase (y gets larger).

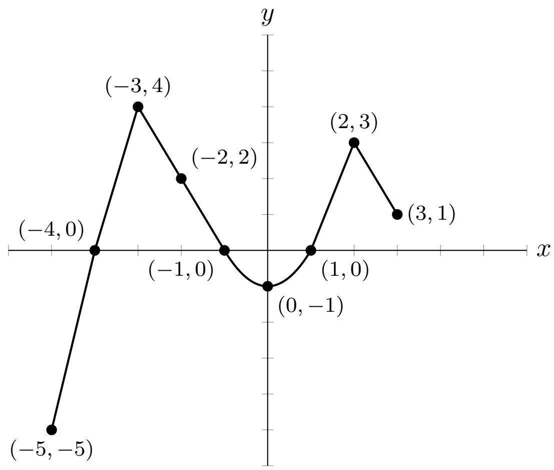
Decreasing: A function is decreasing over an interval if as the inputs decrease (x gets smaller) the outputs decrease (y gets smaller).

Constant: A function is constant over an interval if the output remains the same regardless of the input.

Even: A function is even if $f(-x) = f(x)$ for all x in the domain of f. Graphically, an even function is symmetric about the vertical axis.

Odd: A function is odd if $f(-x) = -f(x)$ for all x in the domain of f. Graphically, an odd function is symmetric about the origin.

1. Given the graph provided, answer all of the following questions.



$y = f(x)$

(a) Find the domain of f

(b) Find the range of f

(c) Find the maximum, if it exists.

(d) Find the minimum, if it exists.

(e) List the local maximums, if any exist.

(f) List the local minimums, if any exist.

(g) List the intervals where f is increasing.

(h) List the intervals where f is decreasing.

(i) Determine $f(-2)$.

(j) Solve $f(x) = 4$.

(k) List the x-intercepts, if any exist.

(l) List the y-intercepts, if any exist.

(m) Find the zeros of f.

(n) Solve $f(x) \geq 0$.

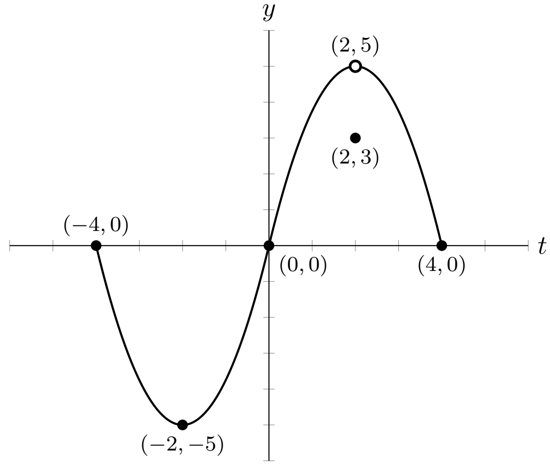
(o) Find the number of solutions to $f(x) = 1$.

(p) Find the number of solutions to $|f(x)| = 1$.

(q) Solve $(x^2 - x - 2)f(x) = 0$.

(r) Solve $(x^2 - x - 2)f(x) > 0$.

2. Given the graph provided, answer all of the following questions.



$y = g(t)$

(a) Find the domain of g.

(b) Find the range of g.

(c) Find the maximum, if it exists.

(d) Find the minimum, if it exists.

(e) List of the local maximums, if any exist.

(f) List the local minimums, if any exist.

(g) List the intervals where g is increasing.

(h) List the intervals where g is decreasing.

(i) Determine $g(2)$.

(j) Solve $g(t) = -5$

(k) List the t-intercepts, if any exists.

(l) List the y-intercepts, if any exist.

(m) Find the zeros of g.

(n) Solve $g(t) \leq 0$.

(o) Find the domain of $G(t) = \dfrac{g(t)}{t+2}$

(p) Solve $\dfrac{g(t)}{t+2} \leq 0$

(q) How many solutions are there to $[g(t)]^2 = 9$?

(r) Does g appear to be even, odd, or neither?