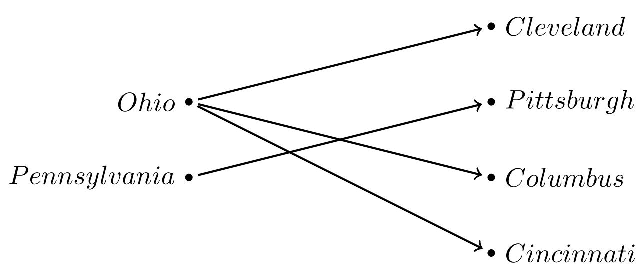
# **5.5** | Relations and Implicit Functions

If you recall way back to section 1.1 (the very first section in the textbook) we define an important restriction on mappings that meet the criteria to be a function. This restriction was that elements from the domain set must be mapped to *one and only one* element of the range set. This makes functions useful for mathematics as functions provide no ambiguity when mapping points. You will never have to question an input between two possible outputs. But what happens if we remove this restriction? Then we no longer have a function, instead we call the mapping a relation.

**Relations**: A relation is a mapping from a set A to a set B. This definition puts not conditions on the actual mapping. Thus, all functions are relations, but not all relations are functions.

Let's look to a very similar example[1] of that which was introduced in worksheet 1.1.




This example fails to be a function. If I provide you with the input (Ohio), you have no way of giving me a clear answer. The mapping can tell me any of the following {Cleveland, Columbus, Cincinnati}. However, this is of course a relation. Relations don't care about the fact that one input happens to have three outputs.

1. Come up with your own mapping diagram. Then ask yourself, this this a function, or a relation? If you drew a function, what could you change to make it a relation? If you drew a relation, what can you change to make it strictly a function?

[1]The reverse was shown in section 1.1, where cities mapped to states. This passes as a function.

**Writing Relations**: When we want to write a relation, our first degree of freedom that we earn is the ability to place the variable $y$ more freely within a function. For example, the following equations are relations and not functions.

- $x = y^2$
- $x = a$
- $x^2 + y^2 = 1$
- $x^2 - y^2 = 1$

However we are still offered much more freedom within relations than simply moving the $y$ value around. We can *set builder* notation to form relations which look even more unique than what we are used to.

**Review: Set Builder Notation**: Recall that set builder notation introduces a variable, and the conditions on said variable. This can come in so many forms that it is hard to generalize, but an example might look like:

$$\{(x, y) \mid (x^2 + y^2 - 1)^3 - x^2y^3 = 0\}$$

Don't worry, this is more complicated of a relation than you will see in this class. If you are curious, you can go to `desmos.com/calculator` and graph this relation to see what it looks like.

**Graphing Relations:** If asked to graph a relation in set builder notation, follow the restrictions on the variable closely based on the form of the coordinate pair you are given. If needed, making a table can help.

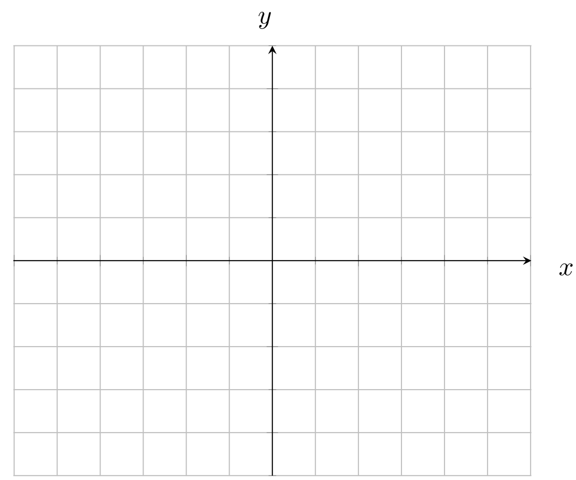
Before we jump into practice problems, let's cover a theorem that is the analogous form of determining if a function is even or odd but for a relation.

**Textbook Theorem 5.9. Testing the Graph of an Equation for Symmetry:** To test the graph of an equation in the $xy$-plane for symmetry:

- about the $x$-axis: substitute $(x, -y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the $x$-axis.
- about the $y$-axis: substitute $(-x, y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the $y$-axis.
- about the origin: substitute $(-x, -y)$ into the equation and simply. If the result is equivalent to the original equation, the graph is symmetric about the origin.

**Note about graphing**: When graphing an inequality, use a solid line to indicate a "$\leq$" or "$\geq$" and a dotted line to indicate a "$<$" or "$>$".

2. **Worked Example:** Graph the relation in the $xy$-plane: $\{(m, 2m) \mid m = 0, \pm 1, \pm 2\}$

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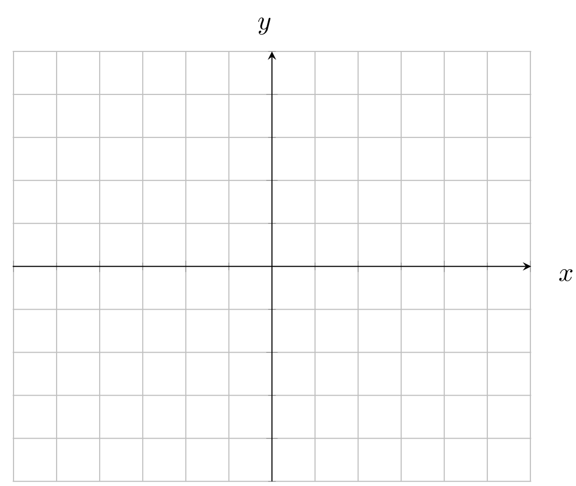
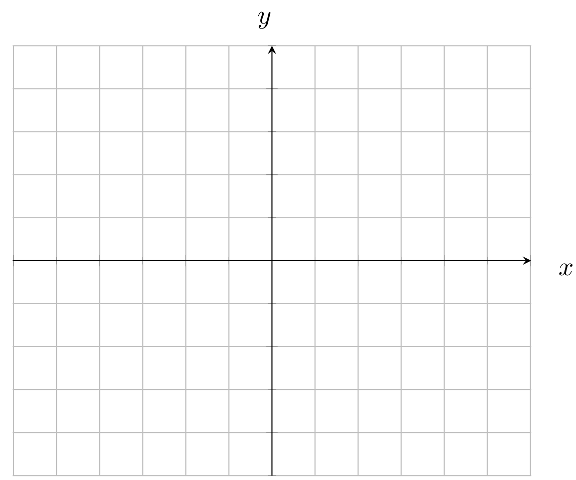
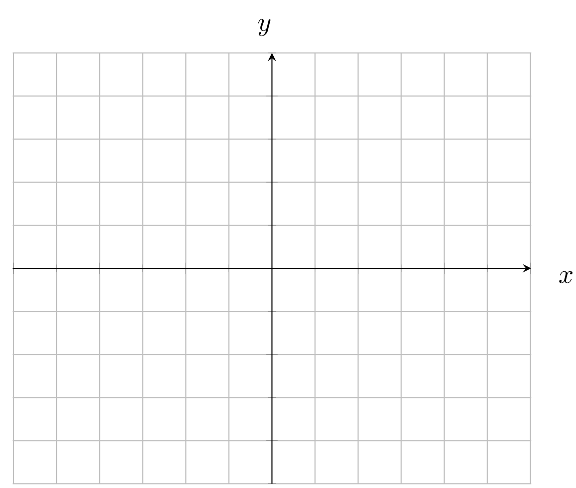
3. Graph the relation in the $xy$-plane: $\{(\sqrt{j}, j) \mid j = 0, 1, 4, 9\}$



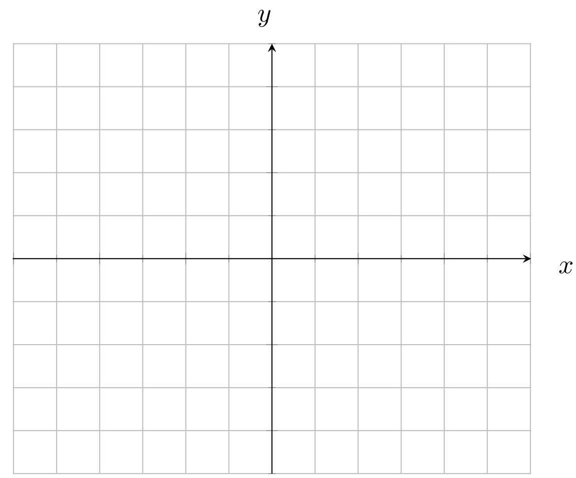
4. Graph the relation in the $xy$-plane: $\{(x, -2) \mid x > -4\}$



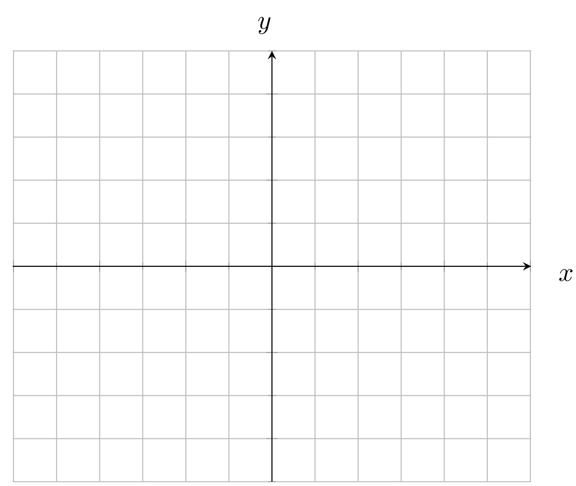

5. Graph the relation in the $xy$-plane: $\{(2, y) \mid y \leq 5\}$



6. Graph the relation in the $xy$-plane: $\{(3, y) \mid -4 \leq y < 3\}$




7. Graph the relation in the $xy$-plane: $\{(x, y) \mid x \leq 3, y < 2\}$

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