6.4 | Equations and Inequalities involving Exponential Functions

Solving Exponential Equations: If we want to solve an exponential equation, we can generally follow these steps. Each equation is different however, so adjustments may need to be made. The best way to improve at solving equations is to practice.

- Isolate the exponential function.
- Take the appropriate logarithm of both sides to reduce the equation. Repeat if necessary.
- Solve the equation like a polynomial.
- Check solutions (keep mind $f(x) = b^x$ will never output a negative value).

> **Problem Solving Tip 1.** If you want to reduce an exponential of base b, you will need to take the logarithm of base b of both sides. This is an extremely important idea that many students forget to utilize. This is an application of the following property: $\log_b(b^x) = x$.

Note About Solutions: In this section, some final solutions may look different that what we are used to. For example, $x = \log_2(3)$ is a perfectly valid solution. To humans, this may look arbitrary, and kind of weird, but we leave it in this form as a decimal approximation would require an infinite amount of digits.[1] We can still simplify this however, a common convention is to simplify to the natural base using the following formula presented from section 6.3:

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

So we might write $x = \log_2(3) = \frac{\ln(3)}{\ln(2)}$ as a final solution.

1. **Worked Example** Solve: $2^{4x} = 8$

Scan the QR code for a video solution

[1] For those curious, $\log_2(3) \approx 1.5849625007211561814537389439478165087598144076924810604557526545\ldots$

2. Solve: $3^{(x-1)} = 27$

3. Solve: $2^{(t^3-t)} = 1$

Problem Solving Tip 2. If both sides of an equation are exponential equations, it can be valuable to rewrite a certain base in terms of another. For example, the two equations 2^x and 8^x appear to be of completely different bases, but if we rewrite 8 as 2^3, we now can see a similarity as we have 2^x and $8^x = (2^3)^x = 2^{3x}$.

4. Solve: $3^{7x} = 81^{4-2x}$

5. $3^{2x} = 5$

6. Solve: $5^t = -2$

7. Solve: $e^{2x} = 2e^x$

8. Solve: $e^{2t} = e^t + 6$

9. Solve: $4^t + 2^t = 12$

10. **Challenge Problem:** Solve: $7^{3+7x} = 3^{4-2x}$

Scan the QR code for a worked solution

Solving Inequalities involving Exponential Functions: When trying to solve an inequality, again every problem is different. However here are some common methods we can try:

- Reduce the exponents using logarithms then solve like a polynomial.
- Factor the expression and make a sign diagram. Keep in mind functions of the form $f(x) = b^x$ will always be positive, functions of the form $f(x) = -b^x$ will always be negative.

11. Solve the inequality: $e^x > 53$

12. Solve the inequality: $2^{(x^3-x)} < 1$

13. Solve the inequality: $25\left(\frac{4}{5}\right)^x \geq 10$

14. Solve the inequality: $e^{-x} - xe^{-x} \geq 0$

15. **Challenge Problem:** Solve the inequality: $(1 - e^t)t^{-1} \leq 0$

Scan the QR code for a worked solution