MATH1300
Selected Challenge Problems
Volume III

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Preface:

These problems are a compilation of problems from the textbook, along with some of my own creations, with the intent to provide a decent challenge to anyone in MATH1300. My goal is that if one is able to *confidently* complete the majority of problems in each section, they should be fairly well prepared for the exam. *Generally* speaking, the problems become more difficult as you move from **A** to **F**, although some students may find earlier problems more difficult and later problems easier. If you find yourself struggling to start *any* problem at all, you may want to go back and review easier questions from the book or my other worksheets before returning to these problems.

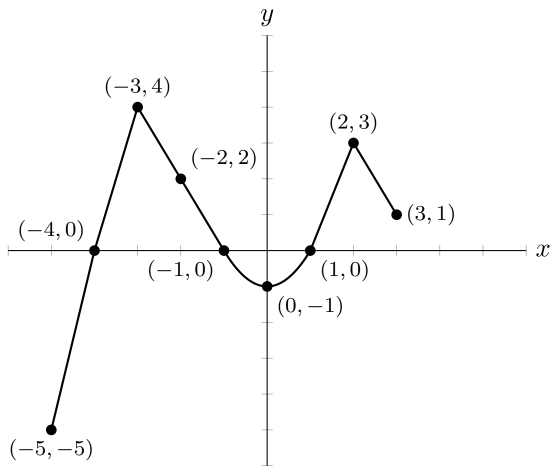
Problems with an **asterisk*** (usually problem **F**) are exceptionally difficult and require a deeper level of introspection into the topic to solve. I would suggest attempting problems with an asterisk only after all preceding problems have been successfully completed. Problems with an asterisk are likely much more difficult than what would show up on an exam, don't worry if you are unable to solve them.

Keep in mind, I have no special insider information on what will actually appear on the exam, and **you should not take this booklet as a representation or study guide of what you will see on your exam.** If you plan to do well on the exam, you **MUST** spend time studying content and problems that appear outside of this booklet.

Roman

5.1

A Given the graph provided, answer all of the following questions.



$y = f(x)$

(a) Find the domain of f

(b) Find the range of f

(c) Find the maximum, if it exists.

(d) Find the minimum, if it exists.

(e) List the local maximums, if any exist.

(f) List the local minimums, if any exist.

(g) List the intervals where f is increasing.

(h) List the intervals where f is decreasing.

(i) Determine $f(-2)$.

(j) Solve $f(x) = 4$.

(k) List the x-intercepts, if any exist.

(l) List the y-intercepts, if any exist.

(m) Find the zeros of f.

(n) Solve $f(x) \geq 0$.

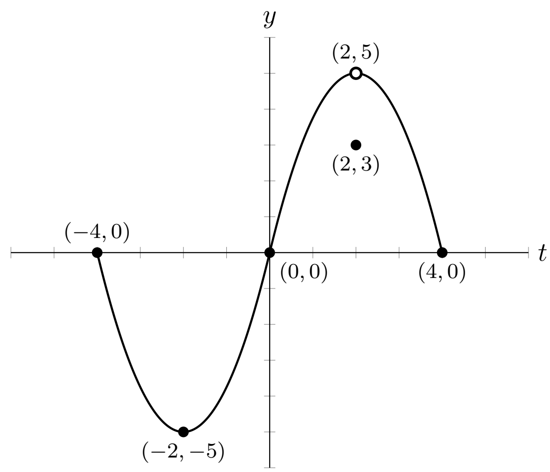
(o) Find the number of solutions to $f(x) = 1$.

(p) Find the number of solutions to $|f(x)| = 1$.

(q) Solve $(x^2 - x - 2)f(x) = 0$.

(r) Solve $(x^2 - x - 2)f(x) > 0$.

B Given the graph provided, answer all of the following questions.



$y = g(t)$

(a) Find the domain of g.

(b) Find the range of g.

(c) Find the maximum, if it exists.

(d) Find the minimum, if it exists.

(e) List of the local maximums, if any exist.

(f) List the local minimums, if any exist.

(g) List the intervals where g is increasing.

(h) List the intervals where g is decreasing.

(i) Determine $g(2)$.

(j) Solve $g(t) = -5$

(k) List the t-intercepts, if any exists.

(l) List the y-intercepts, if any exist.

(m) Find the zeros of g.

(n) Solve $g(t) \leq 0$.

(o) Find the domain of $G(t) = \dfrac{g(t)}{t+2}$

(p) Solve $\dfrac{g(t)}{t+2} \leq 0$

(q) How many solutions are there to $[g(t)]^2 = 9$?

(r) Does g appear to be even, odd, or neither?

5.2

A Let $f(x) = 2x$ and $g(t) = \dfrac{1}{2t+1}$. Compute the indicated value if it exists.

i. $(f+g)(2)$

ii. $\left(\frac{f}{g}\right)(0)$

iii. $(fg)\left(\frac{1}{2}\right)$

B Let f be the function defined by

$$f = \{(-3,4), (-2,2), (-1,0), (0,1), (1,3), (2,4), (3,-1)\}$$

and let g be the function defined by

$$g = \{(-3,-2), (-2,0), (-1,-4), (0,0), (1,-3), (2,1), (3,2)\}$$

Compute the indicated value if it exists.

i. $(g+f)(1)$

ii. $\left(\frac{f}{g}\right)(-2)$

iii. $(gf)(-3)$

C Let $f(x) = x - 1$ and $g(x) = \dfrac{1}{x-1}$, simplify the following expressions.

i. $(f+g)(x)$

ii. $(f-g)(x)$

iii. $(fg)(x)$

iv. $\left(\frac{f}{g}\right)(x)$

D Let $r(x) = \dfrac{3-x}{x+1}$.

i. Find nontrivial[1] functions f and g so that $r = fg$.

E Let $f(x) = -3x + 5$.

i. Find and simplify the difference quotient using the formula: $\frac{f(x+h)-f(x)}{h}$

F Let $f(x) = x - x^2$.

i. Find and simplify the difference quotient using the formula: $\frac{f(x+h)-f(x)}{h}$

[1]Functions like $f(x) = 1$ do not count.

5.3

A Let $f(x) = 4x + 5$ and $g(t) = \sqrt{t}$, compute the following compositions, if any exist.

i. $(g \circ f)(0)$
ii. $(f \circ f)(2)$
iii. $(g \circ f)(-3)$

B Let f be the function defined by

$$f = \{(-3,4),(-2,2),(-1,0),(0,1),(1,3),(2,4),(3,-1)\}$$

and let g be the function defined by

$$g = \{(-3,-2),(-2,0),(-1,-4),(0,0),(1,-3),(2,1),(3,2)\}$$

Compute the indicated value if it exists.

i. $(f \circ g)(3)$
ii. $(f \circ g)(-3)$
iii. $g(f(g(0)))$
iv. $f(f(f(f(f(1)))))$

C Let $f(x) = x^2 - x + 1$ and $g(t) = 3t - 5$. Simplify the indicated composition.

i. $(g \circ f)(x)$
ii. $(f \circ g)(t)$

D Let $f(x) = x^2 - x - 1$ and $g(t) = \sqrt{t - 5}$. Simplify the indicated composition.

i. $(g \circ f)(x)$
ii. $(f \circ g)(t)$

E Let $f(x) = -2x$, $g(t) = \sqrt{t}$, and $h(s) = |s|$. Simplify the indicated composition.

i. $(f \circ g \circ h)(s)$
ii. $(h \circ f \circ g)(t)$
iii. $(g \circ h \circ f)(x)$

F Write $c(x) = \dfrac{x^2}{x^4 + 1}$ as a composition of two or more non-identity functions.

5.4

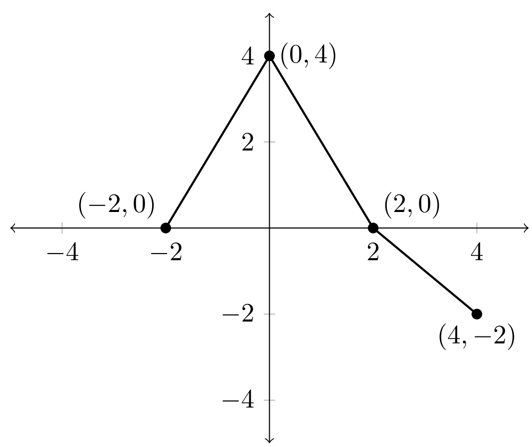
A Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $y = 3f(2x) - 1$.

B Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $y = 5f(2x + 1) + 3$.

C Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $f\left(\frac{7-2x}{4}\right)$.

D Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $\frac{4 - f(3x-1)}{7}$.

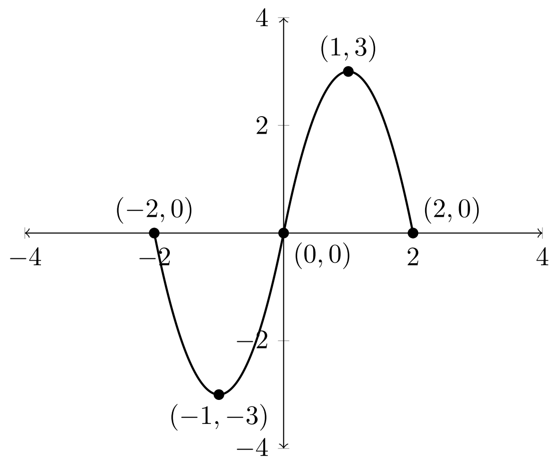
E Given the graph $y = g(t)$



$y = g(t)$

i. Graph the transformation $\frac{1}{2}g(t + 1) - 1$

F Given the graph $y = S(t)$



$y = S(t)$

i. Graph the transformation $y = \frac{1}{2}S(-t + 1) + 1$

5.5

A Graph the indicated relation in the xy-plane.

i. $\{(n, 4-n^2) \mid n = 0, \pm1, \pm2\}$

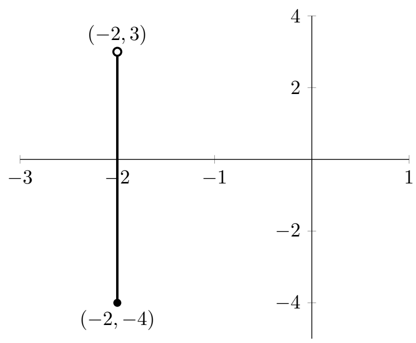
B Graph the indicated relation in the xy-plane.

i. $\{(3, y) \mid -4 \leq y < 3\}$

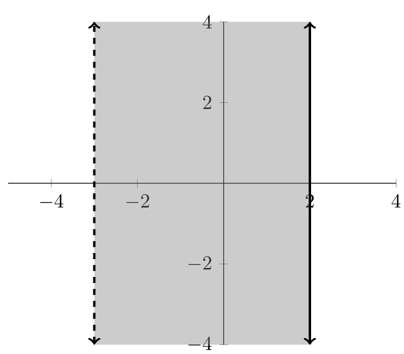
C Graph the indicated relation in the xy-plane.

i. $\{(x, y) \mid x \leq 3, y < 2\}$

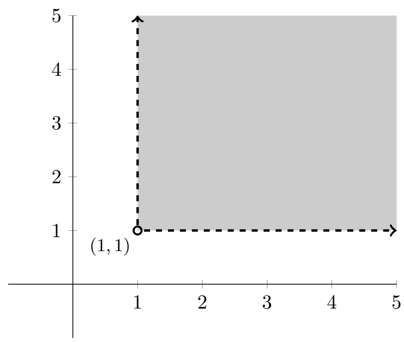
D Describe the given relation using set-builder notation.



E Describe the given relation using set-builder notation.



F Describe the given relation using set-builder notation.



5.6

A Let $f(x) = 2x + 7$ and $g(x) = \dfrac{x-7}{2}$.

i. Graph $f(x)$ and $g(x)$ on a coordinate plane.

ii. Are $f(x)$ and $g(x)$ inverse? Justify your answer.

B Let $g(t)) = \dfrac{t-2}{3} + 4$.

i. Show that $g(t)$ is one-to-one.

ii. Find the inverse of $g(t)$.

C Let $f(x) = \sqrt{3x-1} + 5$.

i. Show that $f(x)$ is one-to-one.

ii. Find the inverse of $f(t)$.

D Let $f(x) = \sqrt[5]{3x-1}$

i. Show that $f(x)$ is one-to-one.

ii. Find $f^{-1}(x)$.

E Let $h(x) = \dfrac{2x-1}{3x+4}$

i. Show that $h(x)$ is one-to-one

ii. Find $h^{-1}(x)$.

F* Under what conditions is $f(x) = mx + b$, $m \neq 0$ its own inverse? Prove your answer.

6.1

A Let $f(x) = 3^x$.

i. Sketch the graph of $f(x)$.

ii. Using transformations, graph $g(x) = 3^{-x} + 2$.

B Let $f(x) = 10^x$

i. Sketch the graph of $f(x)$.

ii. Using transformations, graph $g(x) = 10^{\frac{x+1}{2}} - 20$.

C Let $f(t) = e^t$

i. Sketch the graph of $f(t)$.

ii. Using transformations, graph $g(t) = 8 - e^{-t}$.

D State the domain of $T(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

6.2

A Rewrite the expression: $\log(100) = 2$, so that it does not contain a logarithm.

B Evaluate $\log_2(32)$.

C Evaluate $\log_4(8)$.

D Find the domain of $f(x) = \log_7(t^2 + 9t + 18)$.

E Find the domain of $f(x) = \ln(x^2 + 1)$.

F Find the domain of $g(t) = \ln(7 - t) + \ln(t - 4)$.

6.3

A Expand and simplify: $\ln\left(\frac{\sqrt{z}}{xy}\right)$.

B Expand and simplify: $\ln\left(\sqrt[4]{\frac{xy}{ez}}\right)$.

C Write $\frac{1}{2}\log_3(x) - 2\log_3(y) - \log_3(z)$ as a single logarithm.

D Write $\log_5(x) - 3$ as a single logarithm.

E Write $\log_2(x) + \log_4(x)$ as a single logarithm.

F* With the product rule given, prove the quotient rule and power rule for logarithms.

6.4

A Solve $2^{(t^3-t)} = 1$.

B Solve $3^{7x} = 81^{4-2x}$.

C Solve $e^{2t} = e^t + 6$.

D* Solve $7^{3+7x} = 3^{4-2x}$.

E Solve $e^{-x} - xe^{-x} \geq 0$, write your answer in interval notation.

F Solve $(1 - e^t)t^{-1} \leq 0$, write your answer in interval notation.

6.5

A Solve $10\log\left(\frac{x}{10^{-12}}\right) = 150$.

B Solve $3\ln(t) - 2 = 1 - \ln(t)$.

C Solve $\ln(x+1) - \ln(x) = 3$.

D Solve $\ln(t^2) = (\ln(t))^2$.

E Solve $\frac{1-\ln(t)}{t^2} < 0$, write your answer in interval notation.

F* Solve $\ln(t^2) \leq (\ln(t))^2$, write your answer in interval notation.