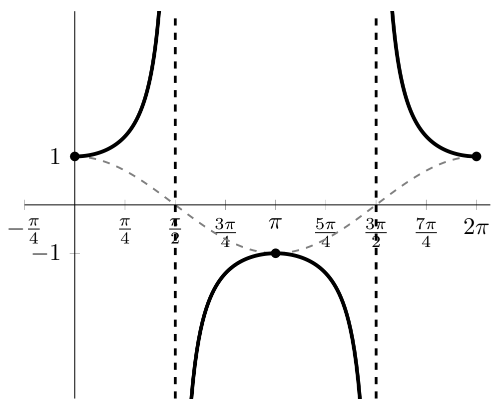
7.5 | Graphs of Secant, Cosecant, Tangent, and Cotangent Functions

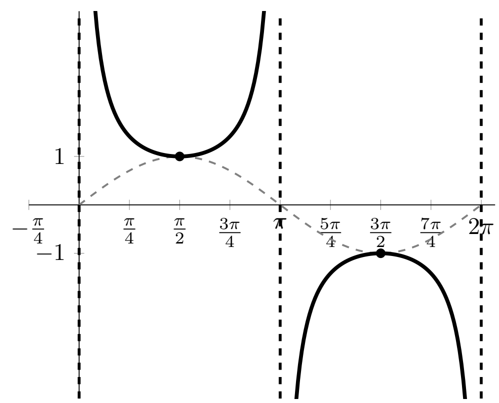
Review of Secant and Cosecant: A brief review of the definitions of secant and cosecant can help put the graphs of these functions into context.

$$\sec(\theta) = \frac{1}{\cos(\theta)} \quad \text{and} \quad \csc(\theta) = \frac{1}{\sin(\theta)}$$

Recall how $\sin(\theta)$ and $\cos(\theta)$ both periodically attain a value of 0. Since we are putting these functions in the denominator, our new functions $\sec(\theta)$ and $\csc(\theta)$ will periodically attain a *vertical asymptote*. In the following representations of $\sec(\theta)$ and $\csc(\theta)$, the corresponding graphs of $\sin(\theta)$ and $\cos(\theta)$ are graphed in a gray dashed line. Notice how the zeroes of sine and cosine align with the asymptotes of secant and cosecant.



$F(t) = \sec(t)$



$G(t) = \csc(t)$

Textbook Theorem 7.10. Properties of the Secant and Cosecant Functions

- The function $F(t) = \sec(t)$
  - has domain $\left\{t \mid t \neq \frac{\pi}{2} + \pi k,\ k \text{ is an integer}\right\}$
  - has range $(-\infty, -1] \cup [1, \infty)$
  - is continuous and smooth on its domain
  - is even
  - has period 2π
- The function $G(t) = \csc(t)$
  - has domain $\{t \mid t \neq \pi k,\ k \text{ is an integer}\}$
  - has range $(-\infty, -1] \cup [1, \infty)$
  - is continuous and smooth on its domain
  - is odd
  - has period 2π

Textbook Theorem 7.11. For $\omega > 0$, the graphs of

$$F(t) = A\sec(\omega t + \phi) + B \quad \text{and} \quad G(t) = A\csc(\omega t + \phi) + B$$

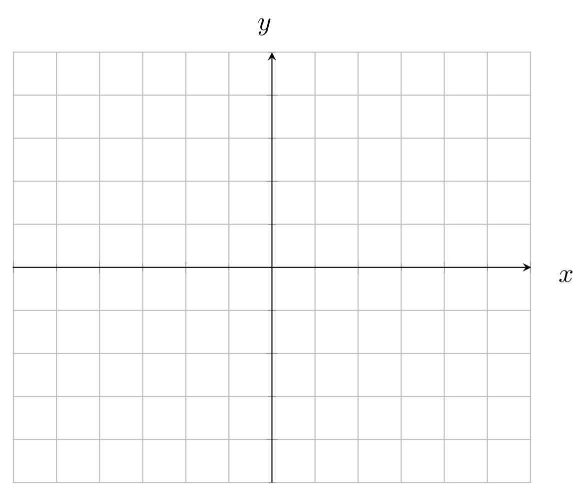
- have period $T = \dfrac{2\pi}{\omega}$
- have phase shift $-\dfrac{\phi}{\omega}$
- have 'baseline' B and have a vertical gap $|A|$ units between the baseline and the graph.[a]

[a] In other words, the range of these functions is $(-\infty, B - |A|] \cup [B + |A|, \infty)$

The Fundamental Cycle: As we did with sine and cosine, when graphing secant and cosecant, we only need to graph one fundamental cycle, as the entire function is periodic and repeats itself forever. You can simply graph the same fundamental cycle as was done with sine and cosine (with a default phase shift of 0 and period of 2π).

Problem Solving Tip 1. Graphing secant and cosecant
To graph either function, lightly graph the corresponding sine or cosine function first using the methods outlined in worksheet 7.3. Then, fill in the secant or cosecant function by matching the asymptotes to the zeros, and the peaks to the valleys and vice versa.

1. **Worked Example:** Graph one cycle of the function: $\sec\left(t - \frac{\pi}{2}\right)$



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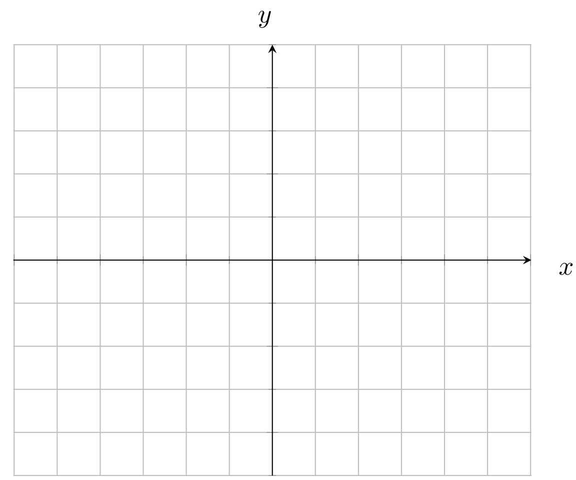
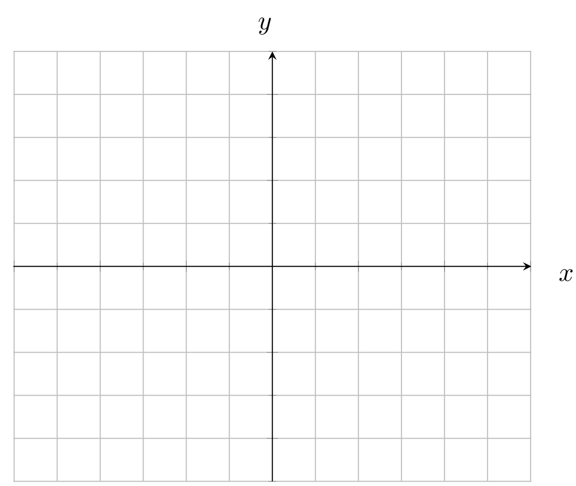
2. Graph one cycle of the function: $\csc(2t - \pi)$

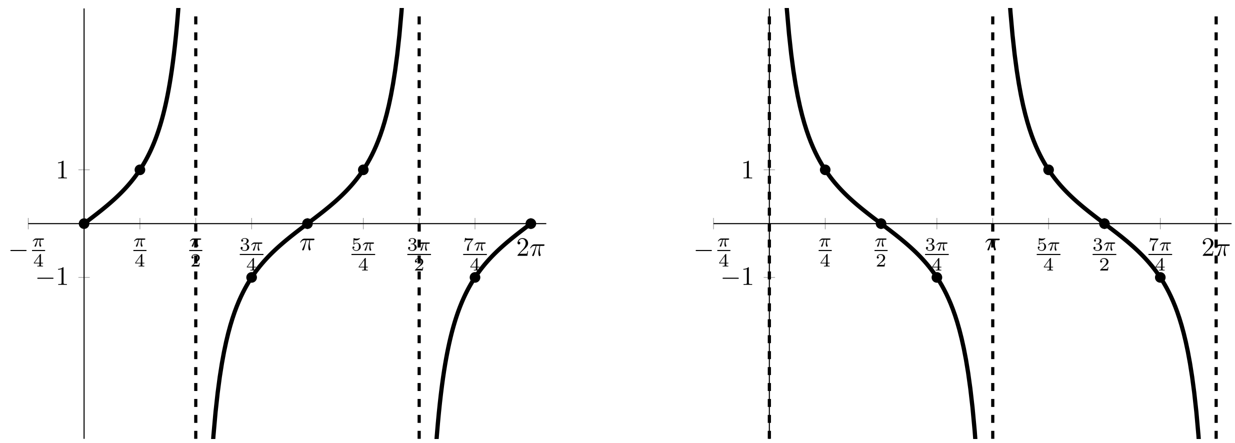


3. Graph one cycle of the function: $\sec(3t - 2\pi) + 4$



Review of Tangent and Cotangent: We will include a reminder of the definitions of tangent and cotangent here, however these graphs are more unique than the other four circular functions and thus have a more unique pattern as well.

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \text{and} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$



$J(t) = \tan(t)$ $\qquad$ $K(t) = \cot(t)$

Textbook Theorem 7.12. Properties of the Tangent and Cotangent Functions

- The function $J(t) = \tan(t)$
  - has domain $\left\{t \mid t \neq \frac{\pi}{2} + \pi k,\ k \text{ is an integer}\right\}$
  - has range $(-\infty, \infty)$
  - is continuous and smooth on its domain
  - is odd
  - has period π
- The function $K(t) = \cot(t)$
  - has domain $\{t \mid t \neq \pi k,\ k \text{ is an integer}\}$
  - has range $(-\infty, \infty)$
  - is continuous and smooth on its domain
  - is odd
  - has period π

Textbook Theorem 7.13. For $\omega > 0$, the functions

$$J(t) = A\tan(\omega t + \phi) + B \quad \text{and} \quad K(t) = A\cot(\omega t + \phi) + B$$

- have period $T = \dfrac{\pi}{\omega}$
- have vertical shift or 'baseline' B
- The phase shift for $y = J(t)$ is $-\dfrac{\phi}{\omega} - \dfrac{\pi}{2\omega}$
- The phase shift for $y = K(t)$ is $-\dfrac{\phi}{\omega}$

The Cycle of Tangent and Cotangent: When graphing a single cycle of tangent and cotangent, remember that the period by default is π, so you only need to graph the start of the function (where the phase shift places it) and then a single cycle which is one period long (again, π by default).

Patterns of the Points of Interest for Tangent and Cotangent Graphs: We can notice some patterns within both functions points of interest that will aid us in graphing. The points of interest in this case are: the asymptotes, the zeros, and the points at -1 and 1.

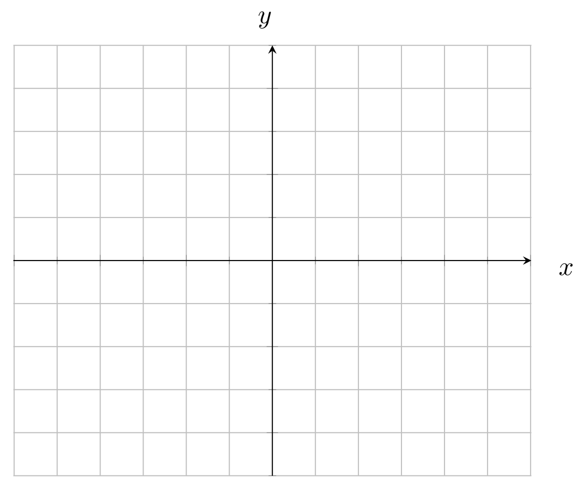
- **Tangent**
  - Starts at an asymptote[1]
  - Is *increasing*
  - Has a point at -1, then 0, and then 1
  - Ends at an asymptote
- **Cotangent**
  - Starts at an asymptote
  - Is *decreasing*
  - Has a point at 1, then 0, and then -1
  - Ends at an asymptote

Problem Solving Tip 2. A clever trick for graphing:
Recall the trick from section 7.3 in which we took advantage of the fact that the points of interest for sine and cosine occur at the 1/4 marks along the graph. This same idea applies to tangent and cotangent. This leads us to the following general method of graphing tangent and cotangent:

Use Theorem 7.13. to determine the starting point and ending point of the cycle for either tangent or cotangent. Then divide the graph into four equal parts. The remaining points of interest lie exactly at the 1/4 mark, the 1/2 mark, and the 3/4 mark.

[1] You might be looking at the graph provided of $\tan(t)$ and argue that it in fact does not start at an asymptote. You are right in noticing this, however the theorem which provides us with the phase shift of tangent is different than that of cotangent. It is specifically designed to place the graph of tangent so that it starts at an asymptote, and so for graphing purposes we may treat tangent as if it starts at an asymptote.

4. **Worked Example:** Graph one cycle of the function: $\tan\left(t - \frac{\pi}{3}\right)$



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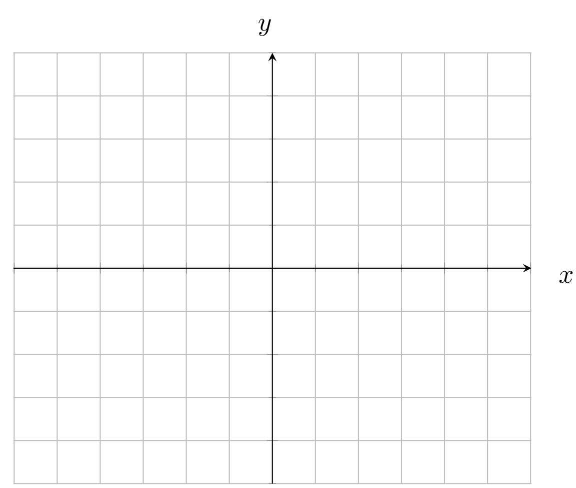
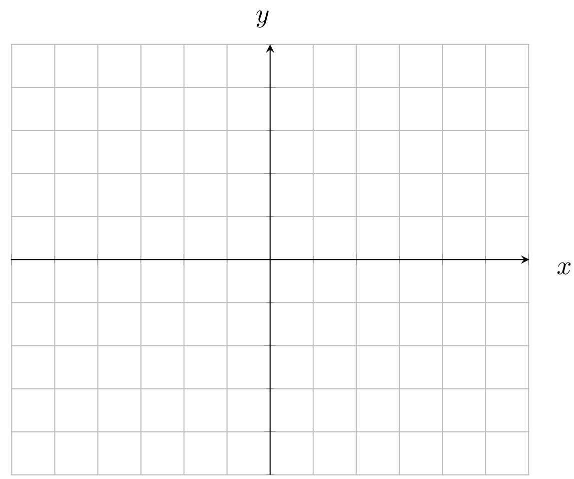
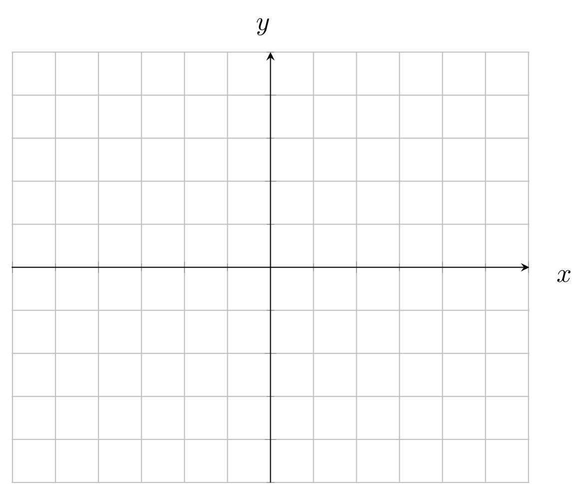
5. Graph one cycle of the function: $2\tan\left(\frac{1}{4}t\right) - 3$



6. Graph one cycle of the function: $\cot\left(t + \frac{\pi}{6}\right)$



7. Graph one cycle of the function: $\frac{1}{3}\cot\left(2t + \frac{3\pi}{2}\right) + 1$



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