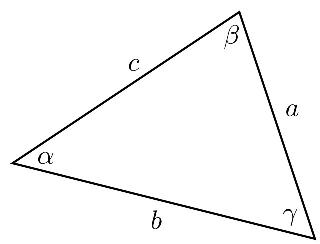
9.1 | The Law of Sines

Practice problems in this worksheet assume the use of a calculator.

In this section, we look at how sine functions can be applied to problems involving trigonometry. Throughout this section, we will refer to an arbitrary triangle with *angle-side opposite pairs*. This implies that a pair (α, a) consists of an angle (α), and then the side of the triangle directly opposite of itself (a). Every triangle comes with three angle-side opposite pairs, and throughout this worksheet we will refer the the three unique angle-side opposite pairs on any triangle as $(\alpha, a), (\beta, b)$ and (γ, c).



Textbook Theorem 9.1. The Law of Sines: Given a triangle with angle-side opposite pairs (α, a), (β, b) and (γ, c), the following ratios hold:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c} \quad \text{or, equivalently,} \quad \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

Textbook Theorem 9.2. Suppose (α, a) and (γ, c) are intended to be angle-side pairs in a triangle where α, a and c are given. Let $h = c\sin(\alpha)$

- If $a < h$, then no triangle exists which satisfies the given criteria.
- If $a = h$, then $\gamma = 90°$ so exactly one (right) triangle exists which satisfies the given criteria.
- If $h < a < c$, then two distinct triangles exist which satisfy the given criteria.
- If $a \geq c$, then γ is acute and exactly one triangle exists which satisfies the given criteria.

For the following practice problems, solve for the remaining angle-side opposite pairs, or determine that the information does not produce a triangle.

1. **Worked Example:** $\alpha = 13°$, $\beta = 17°$, $a = 5$

Scan the QR code for a video solution

2. $\alpha = 73.2°$, $\beta = 54.1°$, $a = 117$

3. $\alpha = 117°$, $a = 35$, $b = 42$

4. $\alpha = 68.7°$, $a = 88$, $b = 92$

5. $\gamma = 74.6°$, $c = 3$, $a = 3.05$

6. $\alpha = 68.7°$, $a = 70$, $b = 90$

Textbook Theorem 9.3. Suppose (α, a), (β, b) and (γ, c) are the angle-side opposite pairs of a triangle. Then the area A enclosed by the triangle is given by

$$A = \frac{1}{2}bc\sin(\alpha) = \frac{1}{2}ac\sin(\beta) = \frac{1}{2}ab\sin(\gamma)$$

That is, the area enclosed by the triangle $A = \frac{1}{2}(\text{the product of two sides})\sin(\text{of the included angle})$.

7. Find the area of a triangle given $\alpha = 13°$, $\beta = 17°$, and $a = 5$.

8. Find the area of a triangle given $\gamma = 53°$, $\alpha = 53°$, and $c = 28.01$.

9. Find the area of a triangle given $a = 50°$, $a = 25$, and $b = 12.5$.