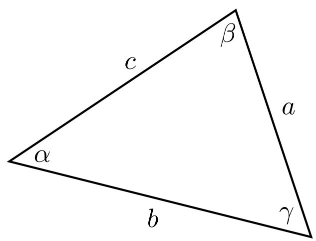
9.2 | The Law of Cosines

Practice problems in this worksheet assume the use of a calculator.

In this section, we introduce a theorem that feels very similar the Pythagorean theorem, except this time we lift the constraint of needing a right triangle by taking advantage of the cosine function. Again, we refer angle-side opposite pairs (α, a), (β, b), and (γ, c) as we did in worksheet 9.1.



Textbook Theorem 9.4. Law of Cosines: Given a triangle with angle-side opposite pairs (α, a), (β, b) and (γ, c), the following equations hold

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha) \qquad b^2 = a^2 + c^2 - 2ac\cos(\beta) \qquad c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

or, solving for the cosine in each equation, we have

$$\cos(\alpha) = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos(\gamma) = \frac{a^2 + b^2 - c^2}{2ab}$$

1. Find the remaining sides and angles if possible: $a = 7, b = 12, \gamma = 59.3°$

2. Find the remaining sides and angles if possible: $a = 104°, b = 25, c = 37$.

3. Find the remaining sides and angles if possible: $a = 153, \beta = 8.2°, c = 153$

4. Find the remaining sides and angles if possible: $a = 1, b = 2, c = 5$

Textbook Theorem 9.5. Heron's Formula: Suppose a, b and c denote the lengths of the three sides of a triangle. Let s be the semiperimeter of the triangle, that is, let $s = \frac{1}{2}(a + b + c)$. Then the area A enclosed by the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

5. Find the area of the triangle with sides $a = 7, b = 10, c = 13$

6. Find the area of the triangle with sides $a = 300, b = 302, c = 48$

7. Find the area of the triangle with sides $a = 5, b = 12, c = 13$