MATH1300
Selected Challenge Problems
Volume IV

SOLUTIONS

Precalculus Peer Assisted Learning

December 5, 2024

Solution Preface:

I make no guarantee that these solutions are free of typos or errors. I have done my best to ensure they are all correct; however, you should always double-check my solutions with another resource if you are unsure whether they are accurate.

Function graphs in this packet are generated by a computer and may not look exactly like the ones you draw. Remember, you are not a computer. The most important skill to develop is your ability to analyze key information about the graph and sketch a relatively accurate picture.

Roman

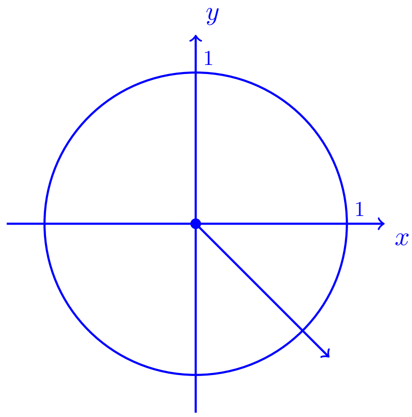
7.1

A Convert 135° into radians.
$\frac{3\pi}{4}$

B Convert $\dfrac{5\pi}{3}$ into degrees.
300°

C Let $\theta = \dfrac{15\pi}{4}$

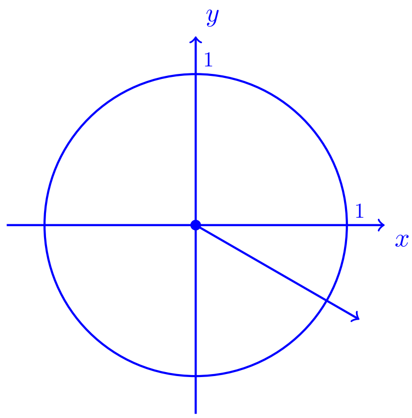
i. Graph θ in standard position.



ii. Give two angles coterminal to θ, one which is positive and one which is negative.
More than one answer, one example is: $\frac{7\pi}{4}, -\frac{\pi}{4}$

D Let $\theta = -\dfrac{13\pi}{6}$

i. Graph θ in standard position.



ii. Give two angles coterminal to θ, one which is positive and one which is negative.
More than one answer, one example is: $\frac{11\pi}{6}, -\frac{\pi}{6}$

7.2

A Given $\theta = \dfrac{3\pi}{4}$

i. Find the value of $\sin(\theta)$.
$\frac{\sqrt{2}}{2}$

ii. Find the value of $\cos(\theta)$.
$-\frac{\sqrt{2}}{2}$

B Find all angles which satisfy the equation: $\sin(\theta) = \dfrac{\sqrt{3}}{2}$
$\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{2\pi}{3} + 2\pi k$ where k is any integer.

C Let θ be an angle in standard position whose terminal side contains the point $P(5, -9)$.

i. Compute $\cos(\theta)$.
$\frac{5\sqrt{106}}{106}$

ii. Compute $\sin(\theta)$.
$-\frac{9\sqrt{106}}{106}$

D Assume $\cos(\theta) = -\dfrac{2}{11}$ with θ in Quadrant III.

i. Find the value of $\sin(\theta)$.
$-\frac{\sqrt{117}}{11}$

E Assume $\sin(\theta) = \dfrac{2\sqrt{5}}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$.

i. Find the value of $\cos(\theta)$.
$-\frac{\sqrt{5}}{5}$

F Draw the unit circle from memory.[1]
Google it ☺

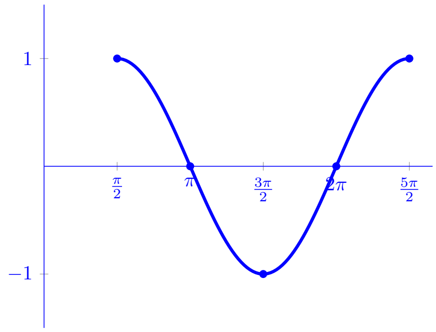
[1] This would not be asked on a test, but you should be able to do this.

7.3

A Let $f(t) = \cos\left(t - \frac{\pi}{2}\right)$

i. State the amplitude, baseline, period, and phase shift of $f(t)$.
Amplitude: 1, Baseline: 0, Period: 2π, Phase Shift: $\frac{\pi}{2}$

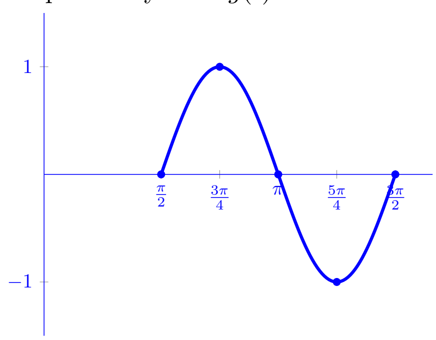
ii. Graph one cycle of $f(t)$.



B Let $g(t) = \sin(2t - \pi)$

i. State the amplitude, baseline, period, and phase shift of $g(t)$.
Amplitude: 1, Baseline: 0, Period: π, Phase Shift: $\frac{\pi}{2}$

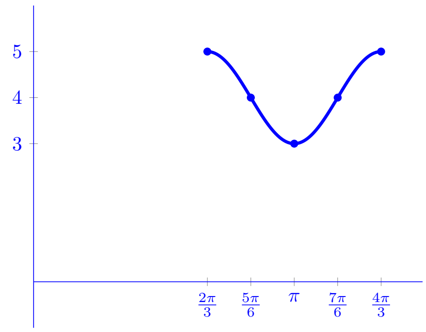
ii. Graph one cycle of $g(t)$.



C Let $h(t) = \cos(3t - 2\pi) + 4$

i. State the amplitude, baseline, period, and phase shift of $h(t)$.
Amplitude: 1, Baseline: 4, Period: $\frac{2\pi}{3}$, Phase Shift: $\frac{2\pi}{3}$

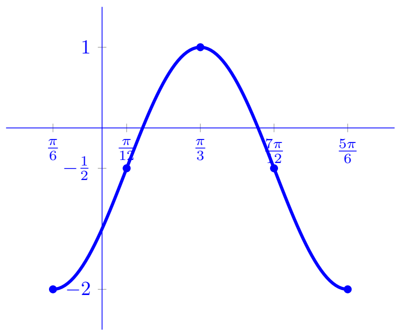
ii. Graph one cycle of $h(t)$.



D Let $q(t) = -\frac{3}{2}\cos\left(2t + \frac{\pi}{3}\right) - \frac{1}{2}$

i. State the amplitude, baseline, period, and phase shift of $q(t)$.
Amplitude: $\frac{3}{2}$, Baseline: $-\frac{1}{2}$, Period: π, Phase Shift: $-\frac{\pi}{6}$

ii. Graph one cycle of $q(t)$.



E* Let S be a collection of sine functions

$$S = \{\sin(\omega_1 x),\, \sin(\omega_2 x),\, \sin(\omega_3 x), \ldots,\, \sin(\omega_n x)\}$$

where no two values of ω are the same. Find a value of x, other than $x = 0$, where all of the sine functions in S equal 0 at the same time.

Proof.

Note that for any arbitrary sine function $\sin(\omega_k x)$ in S, the zeros of the function occur at multiples of $\frac{\pi}{\omega_k}$. So all we need is to find a number that is a multiple of all values of $\frac{\pi}{\omega_k}$. We can use the function $\text{LCM}(a_1, a_2, a_3, \ldots, a_n)$ to denote the least common multiple of n numbers. Define

$$\mathbf{x} = \text{LCM}\left(\frac{\pi}{\omega_1}, \frac{\pi}{\omega_2}, \frac{\pi}{\omega_3}, \ldots, \frac{\pi}{\omega_n}\right)$$

Then $\sin(\omega_1 \mathbf{x}) = \sin(\omega_2 \mathbf{x}) = \sin(\omega_1 \mathbf{x}) = \cdots = \sin(\omega_n \mathbf{x}) = 0$.

□

7.4

A Find the value of $\csc\left(\frac{5\pi}{6}\right)$ if it exists.
2

B Find the value of $\sec\left(-\frac{3\pi}{2}\right)$ if it exists.
undefined

C If it is known that $\sin(\theta) > 0$ but $\tan(\theta) < 0$, in what quadrant does θ lie?
Quadrant II

D Assume $\tan(\theta) = \frac{12}{5}$ with θ in Quadrant III.

i. Find the value of the other five circular functions.
$\sin(\theta) = -\frac{12}{13}, \cos(\theta) = -\frac{5}{13}, \csc(\theta) = -\frac{13}{12}, \sec(\theta) = -\frac{13}{5}, \cot(\theta) = \frac{5}{12}$

E Assume $\cot(\theta) = 2$ with $0 < \theta < \frac{\pi}{2}$

i. Find the value of the other five circular functions.
$\sin(\theta) = \frac{\sqrt{5}}{5}, \cos(\theta) = \frac{2\sqrt{5}}{5}, \tan(\theta) = \frac{1}{2}, \csc(\theta) = \sqrt{5}, \sec(\theta) = \frac{\sqrt{5}}{2}$

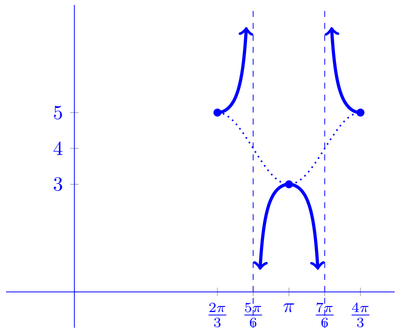
F Find all angles which satisfy the equation $\tan(\theta) = -1$
$\theta = \frac{3\pi}{4} + \pi k$ where k is an integer.

7.5

A Let $f(t) = \sec(3t - 2\pi) + 4$

i. State the period of $f(t)$.
Period: $\frac{2\pi}{3}$

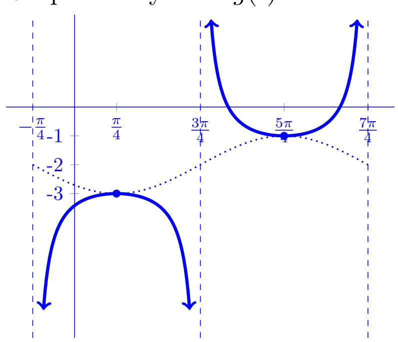
ii. Graph one cycle of $f(t)$.



B Let $g(t) = \csc\left(-t - \frac{\pi}{4}\right) - 2$

i. State the period of $g(t)$.
Period: 2π

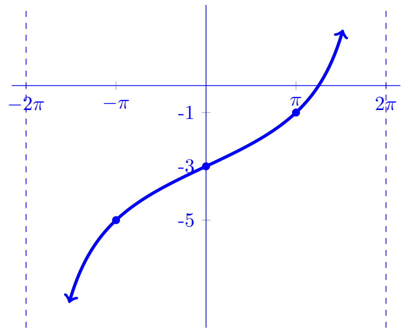
ii. Graph one cycle of $g(t)$.



C Let $r(t) = 2\tan\left(\frac{1}{4}t\right) - 3$

i. State the period of $r(t)$.
Period: 4π

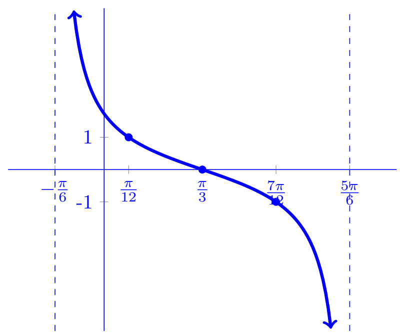
ii. Graph one cycle of $r(t)$.



D Let $s(t) = \cot\left(t + \frac{\pi}{6}\right)$

i. State the period of $s(t)$.
Period: π

ii. Graph one cycle of $s(t)$.



8.1

All identities are true.

A Verify the identity: $\dfrac{\cos(\theta)}{\sin^2(\theta)} = \csc(\theta)\cot(\theta)$

B Verify the identity: $\dfrac{\cos(t)}{1-\sin^2(t)} = \sec(t)$

C Verify the identity: $\tan^3(t) = \tan(t)\sec^2(t) - \tan(t)$

D Verify the identity: $\dfrac{1-\tan(t)}{1+\tan(t)} = \dfrac{\cos(t)-\sin(t)}{\cos(t)+\sin(t)}$

E Verify the identity: $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$

F Verify the identity: $\dfrac{1-\sin(x)}{1+\sin(x)} = (\sec(x)-\tan(x))^2$

8.2

A Find the exact value of $\cos\left(\frac{13\pi}{12}\right)$

$-\frac{\sqrt{6}+\sqrt{2}}{4}$

B Find the exact value of $\sin\left(\frac{\pi}{12}\right)$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

C Let α be a Quadrant IV angle such that $\cos(\alpha) = \frac{\sqrt{5}}{5}$ and let $\frac{\pi}{2} < \beta < \pi$ such that $\sin(\beta) = \frac{\sqrt{10}}{10}$.

i. Find the value of $\cos(\alpha - \beta)$.
$-\frac{\sqrt{2}}{2}$

D Let $0 < \alpha < \frac{\pi}{2}$ such that $\csc(\alpha) = 3$ and let β be a Quadrant II angle such that $\tan(\beta) = -7$.

i. Find the value of $\tan(\alpha + \beta)$.
$\frac{-28+\sqrt{2}}{4+7\sqrt{2}}$ or reformulated $\frac{63-100\sqrt{2}}{41}$

E Verify the identity: $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos(\alpha)\cos(\beta)$.
True

F Verify the identity: $(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$.
True

8.3

A Find the exact value of $\arccos\left(\frac{1}{2}\right)$

$\frac{\pi}{3}$

B Find the exact value of $\text{arccot}(-1)$

$\frac{3\pi}{4}$

C Find the exact value of $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$

$-\frac{\sqrt{2}}{2}$

D Find the exact value of $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$

$\frac{\sqrt{3}}{2}$

E Solve $\sin(\theta) = \frac{7}{11}$

$\theta = \arcsin\left(\frac{7}{11}\right) + 2\pi k$ or $\theta = \pi - \arcsin\left(\frac{7}{11}\right) + 2\pi k$ where k is an integer.

F State the domain of $\arctan(4x)$

$(-\infty, \infty)$

9.1

Chapter 9 is often not included in a final exam.

A Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 13°$, $\beta = 17°$, and $a = 5$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.
$\gamma = 150°, b \approx 6.50, c \approx 11.11$

B Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 73.2°$, $\beta = 54.1°$, and $a = 117$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.
$\gamma = 52.7°, b \approx 99.00, c \approx 97.22$

C Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 95°$, $\beta = 85°$, and $a = 33.33$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.
This information does not produce a triangle.

9.2

Chapter 9 is often not included in a final exam.

A Find the area of the triangle with side lengths, $a = 7$, $b = 10$, and $c = 13$.
$20\sqrt{3}$

B Find the area of the triangle with side lengths, $a = 300$, $b = 302$, and $c = 48$.[2]
$\sqrt{51764375}$

C Find the area of the triangle with side lengths, $a = 5$, $b = 12$, and $c = 13$.
30

[2] Use a calculator.