MATH1300
Selected Challenge Problems
Volume IV

Precalculus Peer Assisted Learning

December 5, 2024

Preface:

These problems are a compilation of problems from the textbook, along with some of my own creations, designed to form multi-step problems that provide a decent challenge to anyone in MATH1300. My goal is that if one is able to complete more than one problem in each section, they should be adequately prepared for the exam. *Generally* speaking, the problems become more difficult as you move from **A** to **F**, although some students may find earlier problems more difficult and later problems easier. If you find yourself struggling to start *any* problem at all, you may want to go back and review easier questions from the book or my other worksheets before returning to these problems. Problems with an **asterisk*** (usually problem **F**) are exceptionally difficult and require a deeper level of introspection into the topic to solve. If you are able to solve a problem with an asterisk, you likely have enough knowledge of the given section to perform well on the exam (no promises).

Keep in mind, I have no special insider information on what will actually appear on the exam, and **you should not take this booklet as a representation of what you will see on your exam.**

7.1

A Convert 135° into radians.

B Convert $\dfrac{5\pi}{3}$ into degrees.

C Let $\theta = \dfrac{15\pi}{4}$

i. Graph θ in standard position.

ii. Give two angles coterminal to θ, one which is positive and one which is negative.

D Let $\theta = -\dfrac{13\pi}{6}$

i. Graph θ in standard position.

ii. Give two angles coterminal to θ, one which is positive and one which is negative.

7.2

A Given $\theta = \dfrac{3\pi}{4}$

i. Find the value of $\sin(\theta)$.

ii. Find the value of $\cos(\theta)$.

B Find all angles which satisfy the equation: $\sin(\theta) = \dfrac{\sqrt{3}}{2}$

C Let θ be an angle in standard position whose terminal side contains the point $P(5, -9)$.

i. Compute $\cos(\theta)$.

ii. Compute $\sin(\theta)$.

D Assume $\cos(\theta) = -\dfrac{2}{11}$ with θ in Quadrant III.

i. Find the value of $\sin(\theta)$.

E Assume $\sin(\theta) = \dfrac{2\sqrt{5}}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$.

i. Find the value of $\cos(\theta)$.

F Draw the unit circle from memory.[1]

[1] This would not be asked on a test, but you should be able to do this.

7.3

A Let $f(t) = \cos\left(t - \frac{\pi}{2}\right)$

i. State the amplitude, baseline, period, and phase shift of $f(t)$.

ii. Graph one cycle of $f(t)$.

B Let $g(t) = \sin(2t - \pi)$

i. State the amplitude, baseline, period, and phase shift of $g(t)$.

ii. Graph one cycle of $g(t)$.

C Let $h(t) = \cos(3t - 2\pi) + 4$

i. State the amplitude, baseline, period, and phase shift of $h(t)$.

ii. Graph one cycle of $h(t)$.

D Let $q(t) = -\frac{3}{2}\cos\left(2t + \frac{\pi}{3}\right) - \frac{1}{2}$

i. State the amplitude, baseline, period, and phase shift of $q(t)$.

ii. Graph one cycle of $q(t)$.

E* Let S be a collection of sine functions

$$S = \{\sin(\omega_0 x),\ \sin(\omega_1 x),\ \sin(\omega_2 x), \ldots,\ \sin(\omega_n x)\}$$

where no two values of ω are the same. Find a value of x, other than $x = 0$, where all of the sine functions in S equal 0 at the same time.

7.4

A Find the value of $\csc\left(\frac{5\pi}{6}\right)$ if it exists.

B Find the value of $\sec\left(-\frac{3\pi}{2}\right)$ if it exists.

C If it is known that $\sin(\theta) > 0$ but $\tan(\theta) < 0$, in what quadrant does θ lie?

D Assume $\tan(\theta) = \frac{12}{5}$ with θ in Quadrant III.

i. Find the value of the other five circular functions.

E Assume $\cot(\theta) = 2$ with $0 < \theta < \frac{\pi}{2}$

i. Find the value of the other five circular functions.

F Find all angles which satisfy the equation $\tan(\theta) = -1$

7.5

A Let $f(t) = \sec(3t - 2\pi) + 4$

i. State the period of $f(t)$.

ii. Graph one cycle of $f(t)$.

B Let $g(t) = \csc\left(-t - \frac{\pi}{4}\right) - 2$

i. State the period of $g(t)$.

ii. Graph one cycle of $g(t)$.

C Let $r(t) = 2\tan\left(\frac{1}{4}t\right) - 3$

i. State the period of $r(t)$.

ii. Graph one cycle of $r(t)$.

D Let $s(t) = \cot\left(t + \frac{\pi}{6}\right)$

i. State the period of $s(t)$.

ii. Graph one cycle of $s(t)$.

8.1

A Verify the identity: $\dfrac{\cos(\theta)}{\sin^2(\theta)} = \csc(\theta)\cot(\theta)$

B Verify the identity: $\dfrac{\cos(t)}{1-\sin^2(t)} = \sec(t)$

C Verify the identity: $\tan^3(t) = \tan(t)\sec^2(t) - \tan(t)$

D Verify the identity: $\dfrac{1-\tan(t)}{1+\tan(t)} = \dfrac{\cos(t)-\sin(t)}{\cos(t)+\sin(t)}$

E Verify the identity: $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$

F Verify the identity: $\dfrac{1-\sin(x)}{1+\sin(x)} = (\sec(x)-\tan(x))^2$

8.2

A Find the exact value of $\cos\left(\dfrac{13\pi}{12}\right)$

B Find the exact value of $\sin\left(\dfrac{\pi}{12}\right)$

C Let α be a Quadrant IV angle such that $\cos(\alpha) = \frac{\sqrt{5}}{5}$ and let $\frac{\pi}{2} < \beta < \pi$ such that $\sin(\beta) = \frac{\sqrt{10}}{10}$.

i. Find the value of $\cos(\alpha - \beta)$.

D Let $0 < \alpha < \frac{\pi}{2}$ such that $\csc(\alpha) = 3$ and let β be a Quadrant II angle such that $\tan(\beta) = -7$.

i. Find the value of $\tan(\alpha + \beta)$.

E Verify the identity: $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos(\alpha)\cos(\beta)$.

F Verify the identity: $(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$.

8.3

A Find the exact value of $\arccos\left(\frac{1}{2}\right)$

B Find the exact value of $\operatorname{arccot}(-1)$

C Find the exact value of $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$

D Find the exact value of $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$

E Solve $\sin(\theta) = \frac{7}{11}$

F State the domain of $\arctan(4x)$

9.1

Chapter 9 is often not included in a final exam.

A Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 13°$, $\beta = 17°$, and $a = 5$.

   i. Does this information produce a triangle? If so, find the remaining values. If not, explain.

B Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 73.2°$, $\beta = 54.1°$, and $a = 117$.

   i. Does this information produce a triangle? If so, find the remaining values. If not, explain.

C Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 95°$, $\beta = 85°$, and $a = 33.33$.

   i. Does this information produce a triangle? If so, find the remaining values. If not, explain.

9.2

Chapter 9 is often not included in a final exam.

A Find the area of the triangle with side lengths, $a = 7$, $b = 10$, and $c = 13$.

B Find the area of the triangle with side lengths, $a = 300$, $b = 302$, and $c = 48$.[2]

C Find the area of the triangle with side lengths, $a = 5$, $b = 12$, and $c = 13$.

[2] Use a calculator.