MATH1300
Selected Challenge Problems
Complete Edition
SOLUTIONS

Precalculus Peer Assisted Learning

Fall 2024

Contents

Solution Preface:

I make no guarantee that these solutions are free of typos or errors. I have done my best to ensure they are all correct; however, you should always double-check my solutions with another resource if you are unsure whether they are accurate.

Function graphs in this packet are generated by a computer and may not look exactly like the ones you draw. Remember, you are not a computer. The most important skill to develop is your ability to analyze key information about the graph and sketch a relatively accurate picture.

Roman

1.1

A Observe the following equation: $2xy = 4$.

i. Does this equation represent y as a function of x?
Yes

ii. If so, write the domain of the equation as set, if not, provide an example where it fails as a function.
$\{x \mid x \in \mathbb{R} \text{ and } x \neq 0\}$

B Observe the set of ordered pairs

$$\{(-3, 9), (1, 1), (3, 1), (0, 0), (-2, 4), (-3, 7), (4, 0)\}$$

i. Does the set of ordered pairs represent a function?
No

ii. If so, write the domain as a set, if not, provide an example where it fails as a function.
$f(-3) = 9 = 7$

C Observe the following data table.

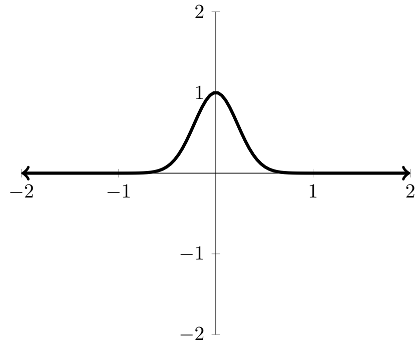
x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

i. Does the given table represent y as a function of x? Explain.
Yes

ii. Write the domain of the table as a set.
$\{-3, -2, -1, 0, 1, 2, 3\}$

iii. Write the range of the table as a set.
$\{0, 1, 2, 3\}$

D Observe the graph

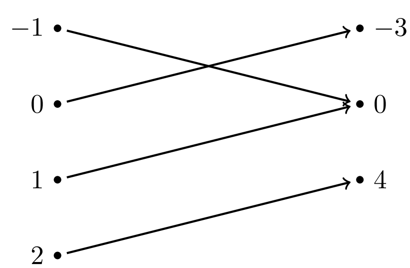


i. Does the graph represent a function? Explain.
Yes, passes vertical line test.

ii. Write the domain of the graph using interval notation.
$(-\infty, \infty)$

iii. Write the range of the graph using interval notation.
$(0, 1]$

E Consider the function f as a mapping diagram shown:



i. Write the domain of f as a set.
$\{-1, 0, 1, 2\}$

ii. Write the range of f as a set.
$\{-3, 0, 4\}$

iii. Find $f(0)$ and solve $f(x) = 0$.
$f(0) = -3$ and $f(x) = 0$ implies $x = -1$ or $x = 1$.

iv. Write f as a set of ordered pairs.
$\{(-1, 0), (0, -3), (1, 0), (2, 4)\}$

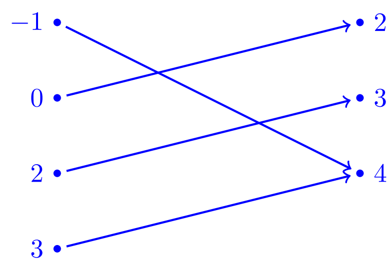
F Let $g = \{(-1,4),(0,2),(2,3),(3,4)\}$

i. Write the domain of g as a set.
$\{-1,0,2,3\}$

ii. Write the range of g as a set.
$\{2,3,4\}$

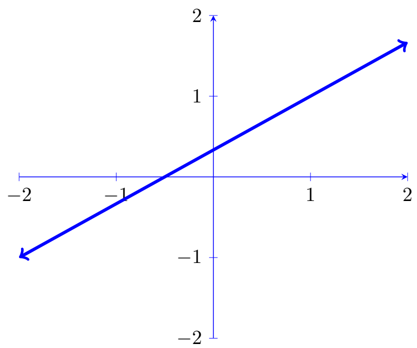
iii. Find $g(0)$ and solve $g(x) = 0$.
$g(0) = 2$ and $g(x) = 0$ has no solution.

iv. Create a mapping diagram for g.



1.2

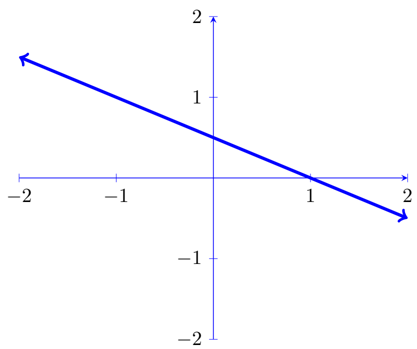
A Graph the function $h(t) = \frac{2}{3}t + \frac{1}{3}$.



i. What is the slope?
$\frac{2}{3}$

ii. State the axis intercepts, if they exist.
$\left(-\frac{1}{2}, 0\right), \left(0, \frac{1}{3}\right)$

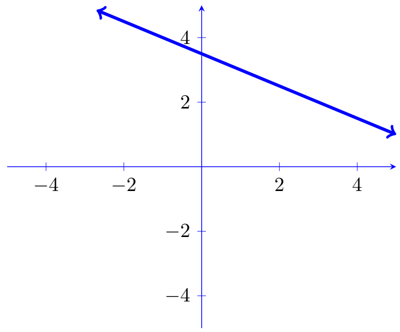
B Graph the function $j(w) = \dfrac{1-w}{2}$



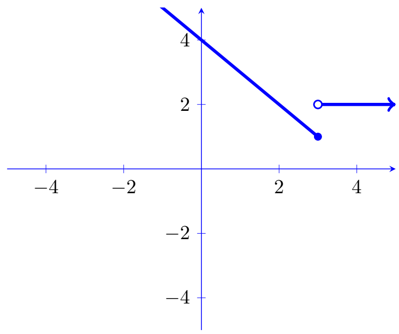
i. What is the slope?
$-\frac{1}{2}$

ii. State the axis intercepts, if they exist.
$\left(0, \frac{1}{2}\right), (1, 0)$

C Find the equation of the function that contains the points $(1, 3)$ and $(3, 2)$.



D Graph the piecewise function $f(x) = \begin{cases} 4 - x & \text{if } x \leq 3 \\ 2 & \text{if } x > 3 \end{cases}$

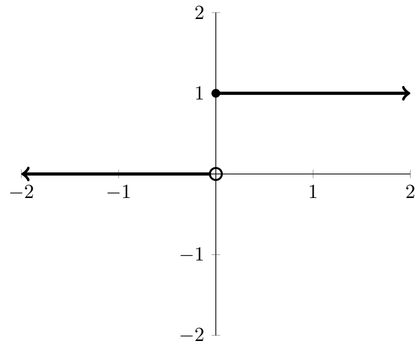


i. Write the domain in interval notation.
$(-\infty, \infty)$

ii. Write the range in interval notation.
$[1, \infty)$

iii. State the axis intercepts, if they exist.
$(0, 4)$

E The unit step function is graphed below:



i. Write the equation $U(t)$ of the unit step function.

$U(t) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 1 \end{cases}$

ii. Write the domain of $U(t)$

$(-\infty, \infty$

iii. Write the range of $U(t)$

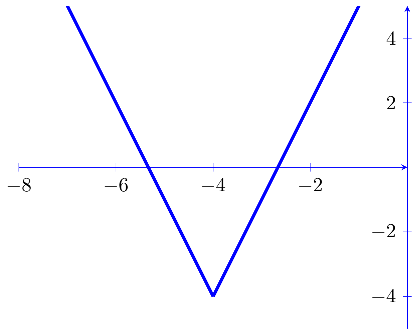
$\{0, 1\}$

F* Explain why the graph of a function $f(x)$ must have at most one y-intercept.

Assume $f(x)$ has more than one y-intercept. Draw a horizontal line on the y-axis, this line intersects the graph more than once, and thus it fails the vertical line test and is not a function.

1.3

A Graph the function $g(t) = 3|t+4| - 4$

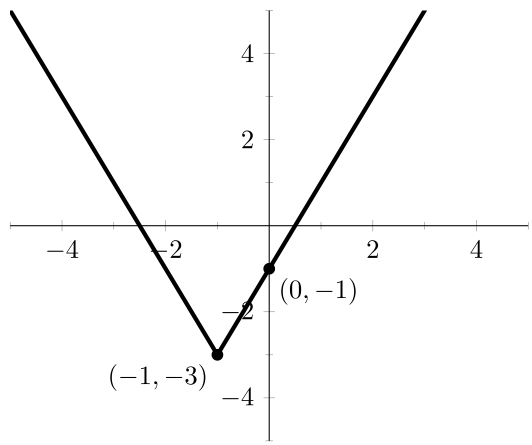


i. Write the domain of $g(t)$ in interval notation.
   $(-\infty, \infty)$

ii. Write the range of $g(t)$ in interval notation.
   $[-4, \infty)$

iii. State the axis intercepts, if they exist.
   $(0, 8)$

B The graph of $F(x)$ is shown below:

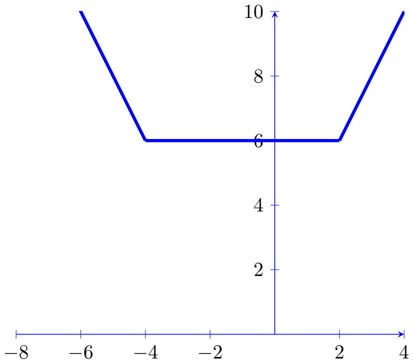


i. Write piecewise function definition of $F(x)$.
   $$F(x) = \begin{cases} -2x - 5 & \text{if } x < -1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$$

ii. State the domain of $F(x)$.
   $(-\infty, \infty)$

iii. State the range of $F(x)$.
   $[-3, \infty)$

C Graph the function $g(x) = |t+4| + |t-2|$.



i. Write the domain of $g(x)$ using interval notation.
$(-\infty, \infty)$

ii. Write the range of $g(x)$ using interval notation.
$[6, \infty)$

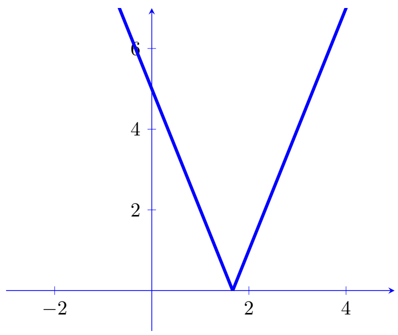
iii. State axis intercepts, if they exist.
$(0, 6)$

D Solve the equation $|3x-2| = |2x+7|$.

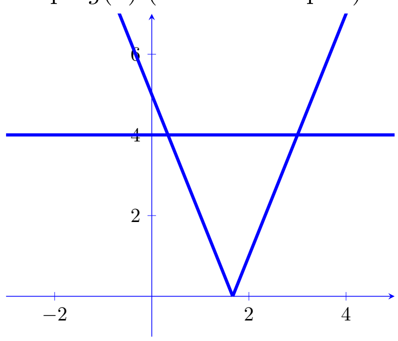
i. Write the solutions as a set.
$\{-1, 9\}$

E Given $f(x) = |3x - 5|$ and $g(x) = 4$

i. Graph $f(x)$.



ii. Graph $g(x)$ (on the same plot).



iii. Solve $f(x) \leq g(x)$. Write your answer in interval notation.
$\left[\frac{1}{3}, 3\right]$

F* Show that if d is a real number with $d > 0$, the solution to $|x - a| < d$ is the interval: $(a - d, a + d)$. That is, an interval centered at a with 'radius' d.

Proof. From the definition of absolute value we know that the distance between x and a must be less than d, we can rephrase this with the relationship $-d < x - a < d$. Adding a to both sides we obtain $-d + a < x < d + a$. With some rearranging we obtain $a - d < x < a + d$ which provides the solution interval $(a - d, a + d)$ for x. □

1.4

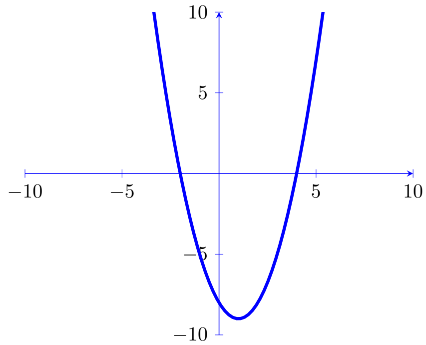
A Let $f(x) = x^2 - 2x - 8$

i. Complete the square on $f(x)$.
$f(x) = (x-1)^2 - 9$

ii. Write the vertex.
$(1, -9)$

iii. Find the axis intercepts.
$(-2, 0), (4, 0)$

iv. Graph $f(x)$.



B Let $h(t) = -3t^2 + 5t + 4$

i. Compute the discriminant of $h(t)$. How many real zeros does $h(t)$ have?
72, this means the function has two positive real roots.

ii. Find the zero(s) of $h(t)$ if they exist, write your solutions as a set.
$\left\{\frac{5-\sqrt{73}}{6}, \frac{5+\sqrt{73}}{6}\right\}$

C Let $g(x) = x^2 - 3x + 9$

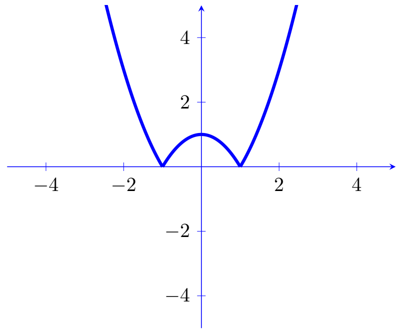
i. Is $g(x)$ factorable?
No

ii. If yes, write $g(x)$ in factored form. If not, explain why.
The discriminant of $g(x)$ is -27, which implies that the function has no real zeros. Therefore it is not factorable.

D Solve the inequality $3x^2 \leq 11x + 4$, write your answer in interval notation.
$\left[-\frac{1}{3}, 4\right]$

E Solve the inequality $5t + 4 \leq 3t^3$, write your answer in interval notation.
$\left(-\infty, \frac{5-\sqrt{73}}{6}\right] \cup \left[\frac{5+\sqrt{73}}{6}, \infty\right)$

F* Graph $f(x) = |1 - x^2|$.



2.1

A Let $g(x) = 3x^5 - 2x^2 + x + 1$

i. Identify the degree of $g(x)$.
5

ii. Identify the leading coefficient of $g(x)$.
3

iii. Identify the leading term of $g(x)$.
$3x^5$

iv. Identify the constant term of $g(x)$.
1

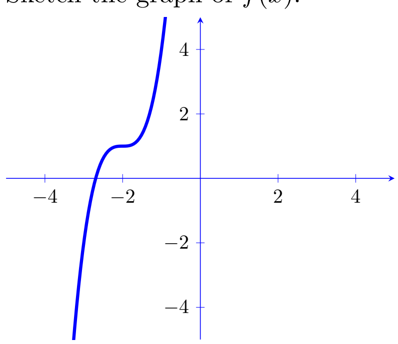
v. Write the end behavior of $g(x)$.
as $x \to \infty$, $f(x) \to \infty$, as $x \to -\infty$, $f(x) \to -\infty$

B Let $f(x) = 3(x + 2)^3 + 1$

i. Write the parent function $P(x)$ for $f(x)$.
$P(x) = x^3$

ii. Pick three points from the parent function $P(x)$ and apply the transformations of $f(x)$ to write three points on the graph of $f(x)$.

iii. Sketch the graph of $f(x)$.



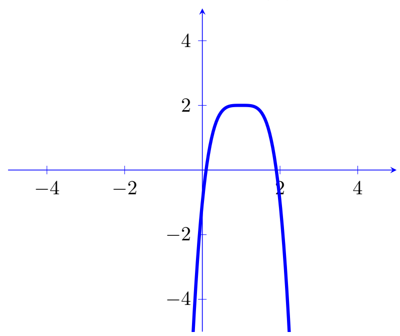
iv. State the domain and range of $f(x)$ using interval notation.
Domain and Range both $(-\infty, \infty)$

C Let $f(x) = 2 - 3(x-1)^4$

i. Write the parent function $P(x)$ for $f(x)$.
$P(x) = x^4$

ii. Pick three points from the parent function $P(x)$ and apply the transformations of $f(x)$ to write three points on the graph of $f(x)$.

iii. Sketch the graph of $f(x)$.



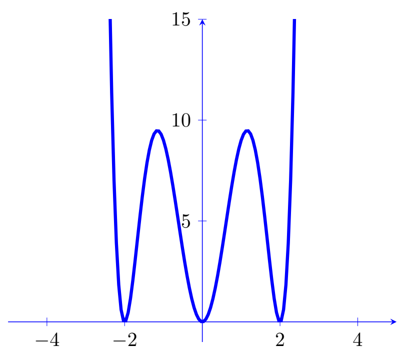
iv. State the domain and range of $f(x)$ using interval notation.
Domain: $(-\infty, \infty)$, Range: $(-\infty, 2]$

D Let $h(t) = t^2(t-2)^2(t+2)^2$

i. List all zeros of $h(t)$ and their corresponding multiplicities.
$t = -2_{m=2}, t = 0_{m=2}, t = 2_{m=2}$

ii. Write the end behavior of $h(t)$.
as $x \to \infty, f(x) \to \infty$, as $x \to -\infty, f(x) \to \infty$.

iii. Sketch a graph of the function $h(t)$.

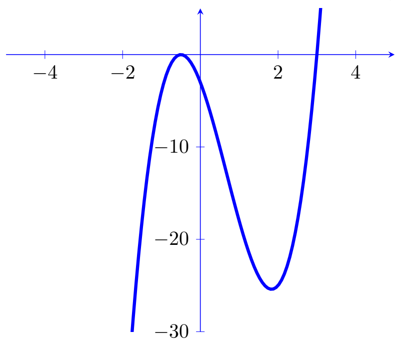


E Let $g(x) = (2x+1)^2(x-3)$

i. List all zeros of $g(x)$ and their corresponding multiplicities.
$x = -\frac{1}{2}_{m=2}, t = 3_{m=1}$

ii. Write the end behavior of $g(x)$.
as $x \to \infty, f(x) \to \infty$, as $x \to -\infty, f(x) \to -\infty$.

iii. Sketch a graph of the function $g(x)$.



F Let $f(x) = (x^2+1)(x-1)$

i. Determine analytically if $f(x)$ is even, odd, or neither.
neither

2.2

A Let $f(z) = 4z^3 + 2z - 3$ and $g(z) = z - 3$

i. Compute $f(z)/g(z)$.
$(4z^3 + 2z - 3) \div (z - 3) = (4x^2 + 12z + 38)\ R11$

ii. Write $f(z)$ as an expression involving $g(z)$, a quotient, and remainder (if it exists).
$(4z^3 + 2z - 3) = (z - 3)(4z^2 + 12z + 38) + 11$

B Let $f(x) = 2x^3 - x + 1$ and $g(x) = x^2 + x + 1$

i. Compute $f(x)/g(x)$.
$(2x^3 - x + 1) \div (x^2 + x + 1) = (2x - 2)\ R(3 - x)$

ii. Write $f(x)$ as an expression involving $g(x)$, a quotient, and remainder (if it exists).
$(2x^3 - x + 1) = (2x - 2)(x^2 + x + 1) + (3 - x)$

C Let $a(x) = x^4 - 6x^2 + 9$ and $b(x) = (x - \sqrt{3})$

i. Compute $a(x)/b(x)$.
$(x^4 - 6x^2 + 9) \div (x - \sqrt{3}) = (x^3 + \sqrt{3}x^2 - 3x - 3\sqrt{3})\ R0$

ii. Write $a(x)$ as an expression involving $b(x)$, a quotient, and remainder (if it exists).
$x^4 - 6x^2 + 9 = (x^3 + \sqrt{3}x^2 - 3x - 3\sqrt{3})(x - \sqrt{3})$

D Let $g(z) = z^3 + 2z^2 - 3z - 6$ be a polynomial function with a known real zero of $c = -2$

i. Find the remaining real zeros of $g(z)$
$z = -2, \sqrt{3}, -\sqrt{3}$

E Let $x^3 - 6x^2 + 11x - 6$ be a polynomial function with a known real zero of $c = 1$

i. Find the remaining real zeros of $g(z)$
$x = 1, 2, 3$

F* Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with the property that $a_n + a_{n-1} + \cdots + a_1 + a_0 = 0$. (That is, the sum of the coefficients and the constant term is 0.)

i. Show that $(x - 1)$ is a factor of $f(x)$.

Proof.

If $(x - 1)$ is a factor then $f(1) = 0$. Plug in $x = 1$ to $f(x)$ to obtain $f(1) = (a_n)1 + (a_{n-1})1 + \cdots + (a_1)1 + (a_0)1$ which simplifies to $a_n + a_{n-1} + \cdots + a_1 + a_0$. By our assumption, $a_n + a_{n-1} + \cdots + a_1 + a_0 = 0$. So $f(1) = 0$ and thus $(x - 1)$ is a factor of f. □

2.3

A Let $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$

i. Use Cauchy's Bound to find an interval containing all possible rational zeros.
$\left[-\frac{4}{3}, \frac{4}{3}\right]$

ii. Use the Rational Zeros Theorem to make a list of possible rational zeros.
$\left\{\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{9}, \pm\frac{1}{12}, \pm\frac{1}{18}, \pm\frac{1}{36},\right\}$

iii. Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros.
2 or 0 positive real zeros, 2 or 0 negative real zeros.

B Let $p(z) = 2z^4 + z^3 - 7z^2 - 3z + 3$

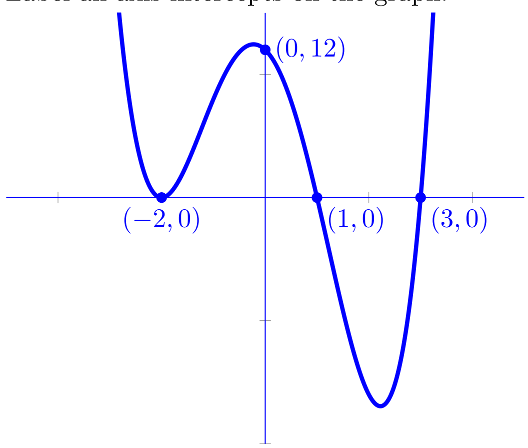
i. Use the Rational Zeros Theorem to list possible roots of the polynomial.
$\left\{\pm 3, \pm 1, \pm\frac{3}{2}, \pm\frac{1}{2}\right\}$

ii. Write the polynomial in factored form.
$(2z-1)(z+1)(z^2-3)$

C Let $g(x) = x^4 - 9x^2 - 4x + 12$

i. Sketch the graph of $g(x)$.

ii. Label all axis intercepts on the graph.



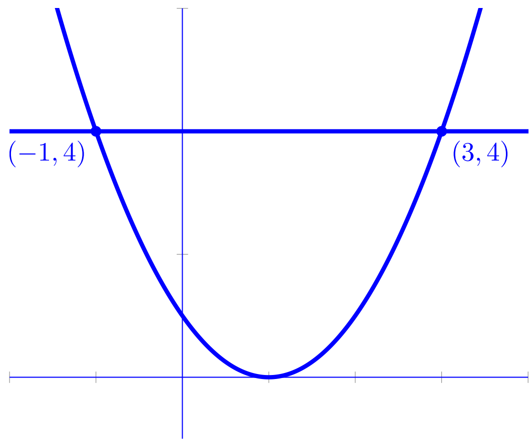
iii. Write the end behavior of $g(x)$.
as $x \to -\infty$, $f(x) \to \infty$
as $x \to \infty$, $f(x) \to \infty$

D Solve the following equation: $x^3 + x^2 = \dfrac{11x + 10}{3}$

$x = -2, \dfrac{3 \pm \sqrt{69}}{6}$

E Let $f(x) = (x-1)^2$ and $g(x) = 4$

i. Graph $f(x)$ and $g(x)$ on the same coordinate plane.



ii. Solve the inequality $f(x) \geq g(x)$ graphically.
$(-\infty, -1] \cup [3, \infty)$

iii. Solve the inequality $f(x) \geq g(x)$ algebraically and verify that it matches the solution found in part (ii).
$(-\infty, -1] \cup [3, \infty)$

F Solve the inequality: $\dfrac{x^3 + 20x}{8} \geq x^2 + 2$, express your answer in interval notation.
$\{2\} \cup [4, \infty)$

3.1

A Let $p(x) = 9x^3 + 5$ and $q(x) = 2x - 3$

i. Divide $p(x) \div q(x)$ using synthetic division or long division.
Synthetic division will make dealing with the fractions in this problem easier.

ii. Write $p(x)$ in the form of $d(x)q(x) + r(x)$.
$(9x^3 + 5) = (2x - 3)\left(\frac{9}{2}x^2 + \frac{27}{4}x + \frac{81}{8}\right) + \frac{283}{8}$

B Let $p(x) = 4x^2 - x - 23$ and $q(x) = x^2 - 1$

i. Divide $p(x) \div q(x)$ using synthetic division or long division.
Must use long division as synthetic division will not work for non linear divisors.

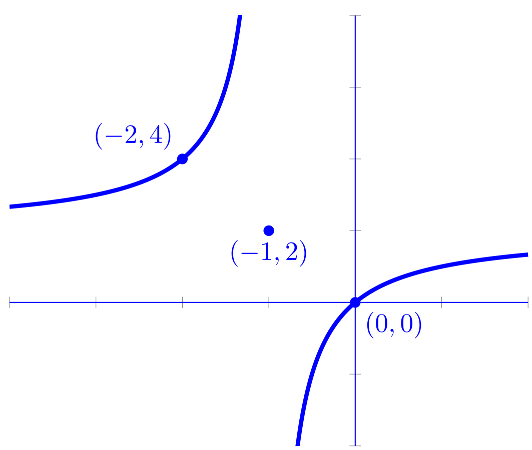
ii. Write $p(x)$ in the form of $d(x)q(x) + r(x)$.
$4x^2 - x - 23 = 4(x^2 - 1) + (-x - 19)$

C Let $h(x) = \dfrac{2x}{x+1}$.

i. Write $h(x)$ in the form $\dfrac{a}{x-h} + k$.
Use division to obtain $h(x) = 2 - \dfrac{2}{x+1}$

ii. Write the parent function $P(x)$ of $h(x)$.
$P(x) = \dfrac{1}{x}$

iii. Track at least two points and the asymptotes from $P(x)$ and use them to graph $h(x)$.
Choose sample points $\{(-1, -1), (1, 1)\}$ and track $(0, 0)$ for asymptotes.



D Let $r(x) = \dfrac{x^2 - x - 12}{x^2 + x - 6}$

i. Identify any holes in the graph of $r(x)$.
   $\left(-3, \frac{7}{5}\right)$

ii. Identify any vertical asymptotes in the graph of $r(x)$.
   $x = 2$

iii. State the domain of $r(x)$.
   $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

E Let $f(x) = \dfrac{x^3 + 2x^2 + x}{x^2 - x - 2}$

i. Identify any holes in the graph of $f(x)$.
   $(-1, 0)$

ii. Identify any vertical asymptotes in the graph of $f(x)$.
   $x = 2$

iii. State the domain of $f(x)$.
   $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

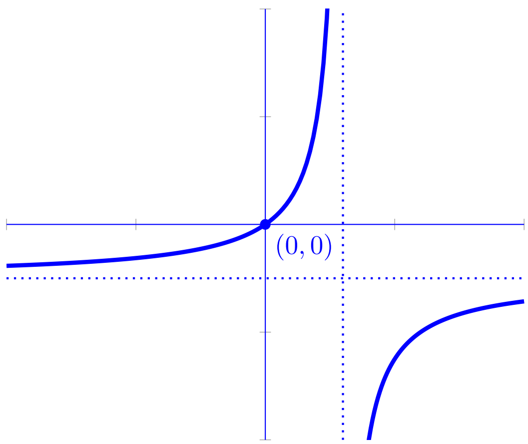
F* Let $u(x)$ be a function defined only on the positive real numbers. Let $v(x) = (x - a)(x + b)$ with $0 < a < b$.

i. State the domain of $w(x) = \dfrac{u(x)}{v(x)}$
   $(0, a) \cup (a, \infty)$

3.2

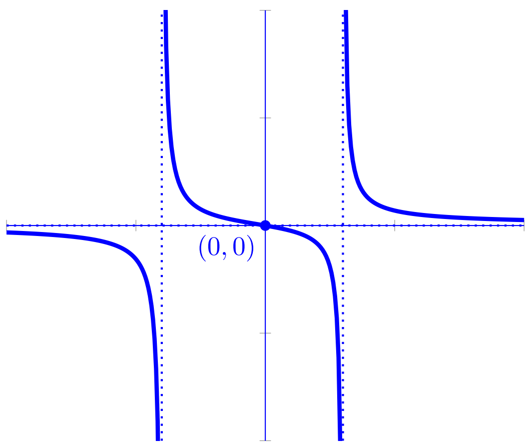
A Let $f(x) = 5x(6-2x)^{-1}$

i. Sketch the graph of $f(x)$. Label all asymptotes, holes, and zeros.



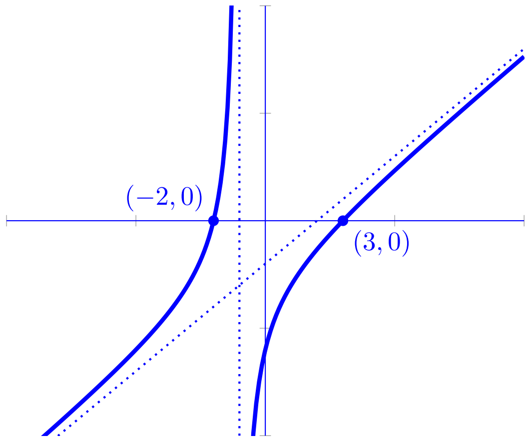
B Let $a(x) = \dfrac{x}{x^2+x-12}$

i. Sketch the graph of $a(x)$. Label all asymptotes, holes, and zeros.



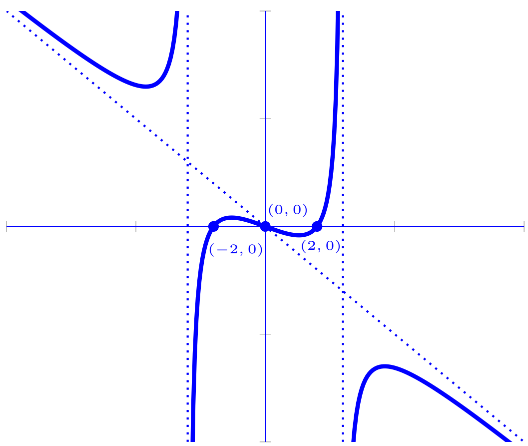
C Let $r(t) = \dfrac{t^2-t-6}{t+1}$

i. Sketch the graph of $r(t)$. Label all asymptotes, holes, and zeros.



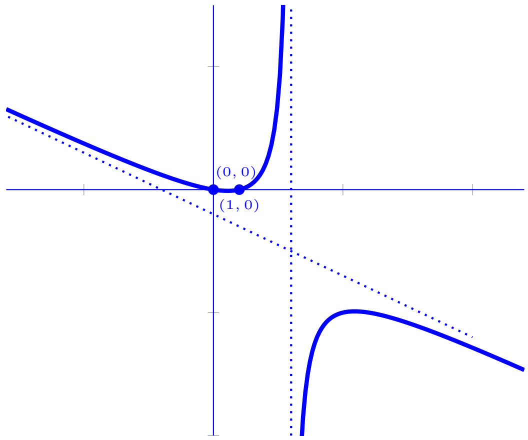
D Let $f(x) = \dfrac{5x}{9 - x^2} - x$

i. Sketch the graph of $f(x)$. Label all asymptotes, holes, and zeros.



E Let $r(z) = -z - 2 + \dfrac{6}{3-z}$

i. Sketch the graph of $r(z)$. Label all asymptotes, holes, and zeros.



F* Let $p(x) = 2x^3 + 5x^2 + 4x + 3$ and $q(x) = 2x + 1$

i. Does $r(x) = \dfrac{p(x)}{q(x)}$ have a horizontal or slant asymptote?
Neither.

ii. Divide $p(x) \div q(x)$ and ignore the remainder. What does this suggest about the (non vertical) asymptote of $r(x)$?
Dividing and ignoring the remainder obtains: $x^2 + 2x + 1$. This suggests that the asymptote of $r(x)$ is a *parabola*.

iii. Assume $a(x)$ is a fourth degree polynomial, and $b(x)$ is a linear. Assuming $b(x)$ is not a factor of $a(x)$, what might the (non vertical) asymptote of $f(x) = \dfrac{a(x)}{b(x)}$ look like?
Dividing a degree four polynomial by a linear term yields a third degree polynomial. So the asymptote of $f(x)$ would be a *cubic function*.

3.3

A Solve $\dfrac{3x-1}{x^2+1} = 1$.
$x = 1, 2$

B Solve $\dfrac{1}{t+3} + \dfrac{1}{t-3} = \dfrac{t^2-3}{t^2-9}$.
$t = -1$

C Solve $\dfrac{4t}{t^2+4} \geq 0$.
$[0, \infty)$

D Solve $\dfrac{2t+6}{t^2+t-6} < 1$.
$(-\infty, -3) \cup (-3, 2) \cup (4, \infty)$

E Solve $\dfrac{3z-1}{z^2+1} \leq 1$.
$(-\infty, 1] \cup [2, \infty)$

F* Solve $\dfrac{2x^2-5x+4}{3x^2+1} < 0$, justify your answer.
$3x^2+1$ is always positive, use the discriminant and vertex form to show that $2x^2-5x+4$ is also always positive, so there are no solutions.

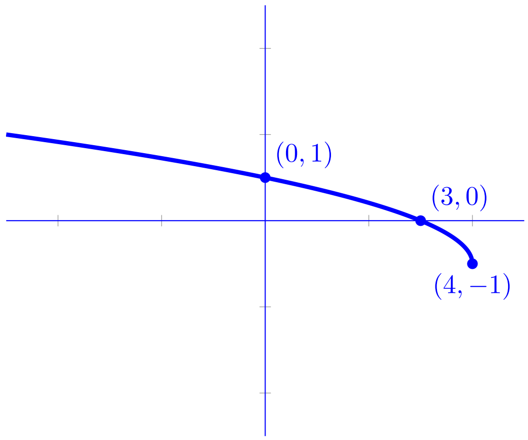
4.1

A Let $f(x) = \sqrt{4-x} - 1$

i. Write the parent function $P(x)$ for f.
$P(x) = \sqrt{x}$

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

Track $(0,0), (1,1)$, and $(4,2)$

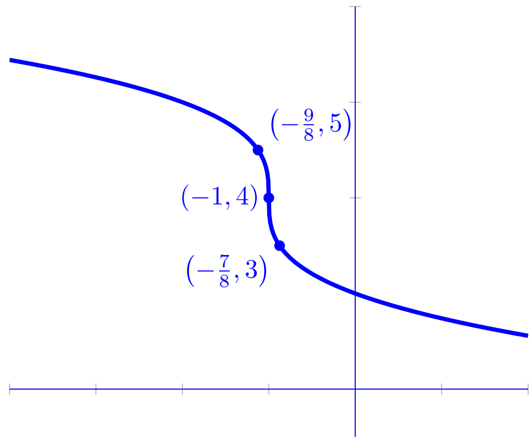


B Let $f(x) = -\sqrt[3]{8x+8} + 4$

i. Write the parent function $P(x)$ for f.
$P(x) = \sqrt[3]{x}$

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

Track $(-1, -1), (0, 0)$, and $(1, 1)$

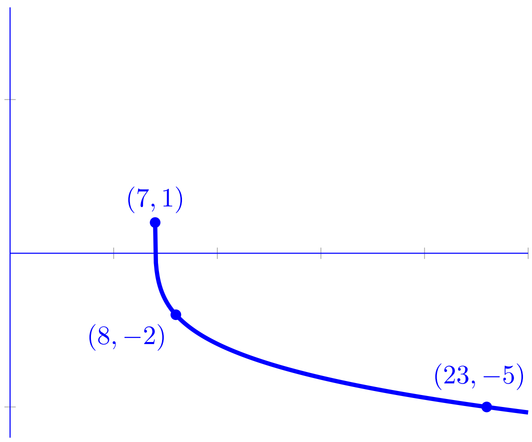


C Let $f(x) = -3\sqrt[4]{x-7} + 1$

i. Write the parent function $P(x)$ for f.
$P(x) = \sqrt[4]{x}$

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

Track $(0, 0), (1, 1)$, and $(16, 2)$



D Let $d(x) = \dfrac{5x}{\sqrt[3]{x^3+8}}$

i. State the domain of $d(x)$.
$(-\infty, -2) \cup (-2, \infty)$

E Let $z(x) = \sqrt{x(x+5)(x-4)}$

i. State the domain of $z(x)$.
$[-5, 0] \cup [4, \infty)$

F Let $c(x) = \sqrt[6]{\dfrac{x^2+x-6}{x^2-2x-15}}$

i. State the domain of $c(x)$.
$(-\infty, -3) \cup (-3, 2] \cup (5, \infty)$

4.2

A Let $c(x) = x^{\frac{4}{7}}$

i. List the intervals where $c(x)$ is increasing (if any exist).
$(0, \infty)$

ii. List the intervals where $c(x)$ is decreasing (if any exist).
$(-\infty, 0)$

iii. List the intervals where $c(x)$ is concave up (if any exist).
No intervals exist.

iv. List the intervals where $c(x)$ is concave down (if any exist).
$(-\infty, 0) \cup (0, \infty)$

B Let $b(t) = t^{\frac{10}{4}}$

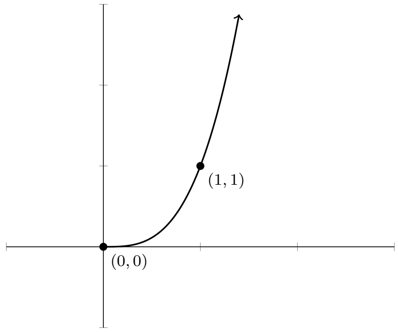
i. List the intervals where $c(x)$ is increasing (if any exist).
$(0, \infty)$

ii. List the intervals where $c(x)$ is decreasing (if any exist).
No intervals exist.

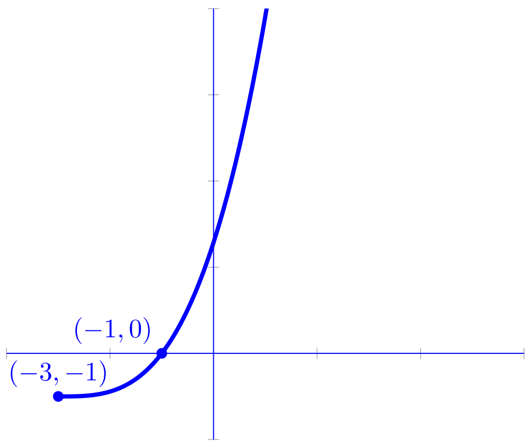
iii. List the intervals where $c(x)$ is concave up (if any exist).
$(0, \infty)$

iv. List the intervals where $c(x)$ is concave down (if any exist).
No intervals exist.

C The graph $g(t) = t^{\pi}$ is shown (where $\pi \approx 3.1415\ldots$).



i. Track the points provided on $g(t)$ to graph $G(t) = \left(\frac{t+3}{2}\right)^{\pi} - 1$



D Let $f(x) = x^{\frac{3}{2}}(x-7)^{\frac{1}{3}}$

i. State the domain of $f(x)$.
$[0, \infty)$

E Let $g(t) = t^{\frac{3}{2}}(t-2)^{-\frac{1}{2}}$

i. State the domain of $f(x)$.
$(2, \infty)$

F* Let $g(t) = 4t(9-t^2)^{-\sqrt{2}}$

i. State the domain of $g(t)$.
$(-3, 3)$

4.3

A Solve the equation $2x + 1 = (3 - 3x)^{\frac{1}{2}}$
$x = \frac{1}{4}$

B Solve the equation $(2x + 1)^{\frac{1}{2}} = 3 + (4 - x)^{\frac{1}{2}}$
$x = 4$

C Solve the equation $2t^{\frac{1}{3}} = 1 - 3t^{\frac{2}{3}}$
$t = -1, \frac{1}{27}$

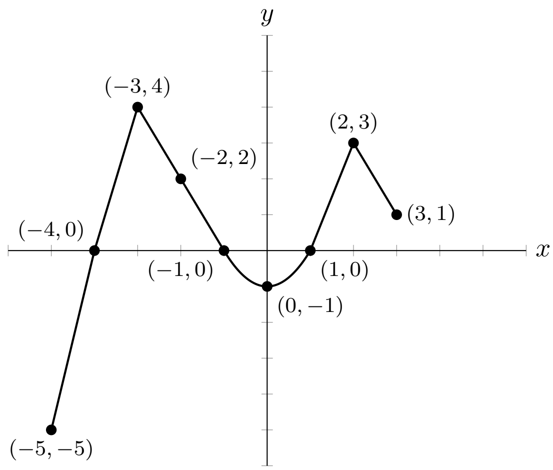
D Solve the inequality $\sqrt[3]{x} > x$, express your answer in interval notation.
$(-\infty, -1) \cup (0, 1)$

E Solve the inequality $(2 - 3x)^{\frac{1}{3}} > 3x$, express your answer in interval notation.
$\left(-\infty, \frac{1}{3}\right)$

F Solve the inequality $3(x - 1)^{\frac{1}{3}} + x(x - 1)^{-\frac{2}{3}} \geq 0$, express your answer in interval notation.
$\left[\frac{3}{4}, 1\right) \cup (1, \infty)$

5.4

A Given the graph provided, answer all of the following questions.



$y = f(x)$

(a) Find the domain of f
$[-5, 3]$

(b) Find the range of f
$[-5, 4]$

(c) Find the maximum, if it exists.
$f(-3) = 4$

(d) Find the minimum, if it exists.
$f(-5) = -5$

(e) List the local maximums, if any exist.
$\{(-3, 4), (2, 3)\}$

(f) List the local minimums, if any exist.
$(0, -1)$

(g) List the intervals where f is increasing.
$[-5, -3] \cup [0, 2]$

(h) List the intervals where f is decreasing.
$[-3, 0] \cup [2, 3]$

(i) Determine $f(-2)$.
$f(-2) = 2$

(j) Solve $f(x) = 4$.
$x = -3$

(k) List the x-intercepts, if any exist.
$\{(-4, 0), (-1, 0), (1, 0)\}$

(l) List the y-intercepts, if any exist.
$(0, -1)$

(m) Find the zeros of f.
$\{-4, -1, 1\}$

(n) Solve $f(x) \geq 0$.
$[-4, -1] \cup [1, 3]$

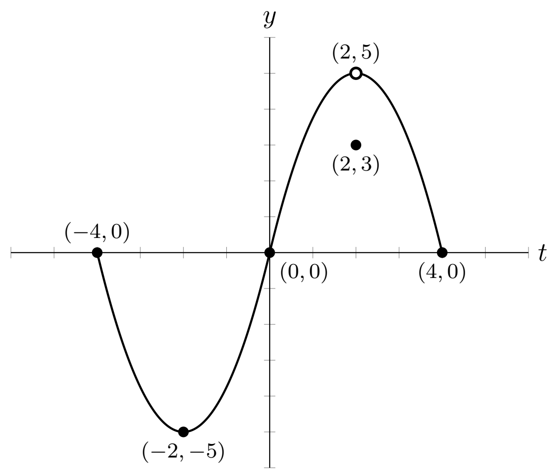
(o) Find the number of solutions to $f(x) = 1$.
4 solutions

(p) Find the number of solutions to $|f(x)| = 1$.
6 solutions

(q) Solve $(x^2 - x - 2)f(x) = 0$.
$x = \{-4, -1, 1, 2\}$

(r) Solve $(x^2 - x - 2)f(x) > 0$.
$(-4, -1) \cup (-1, 1) \cup (2, 3)$

B Given the graph provided, answer all of the following questions.



$y = g(t)$

(a) Find the domain of g.
$[-4, 4]$

(b) Find the range of g.
$[-5, 5)$

(c) Find the maximum, if it exists.
none

(d) Find the minimum, if it exists.
$g(-2) = -5$

(e) List of the local maximums, if any exist.
none

(f) List the local minimums, if any exist.
$\{(-2, -5), (2, 3)\}$

(g) List the intervals where g is increasing.
$[-2, 2)$

(h) List the intervals where g is decreasing.
$[-4, -2] \cup (2, 4]$

(i) Determine $g(2)$.
$g(2) = 3$

(j) Solve $g(t) = -5$
$t = -2$

(k) List the t-intercepts, if any exists.
$\{(-4, 0), (0, 0), (4, 0)\}$

(l) List the y-intercepts, if any exist.
$(0, 0)$

(m) Find the zeros of g.
$\{-4, 0, 4\}$

(n) Solve $g(t) \leq 0$.
$[-4, 0] \cup \{4\}$

(o) Find the domain of $G(t) = \dfrac{g(t)}{t+2}$
$[-4, -2) \cup (-2, 4]$

(p) Solve $\dfrac{g(t)}{t+2} \leq 0$
$\{-4\} \cup (-2, 0] \cup \{4\}$

(q) How many solutions are there to $[g(t)]^2 = 9$?
5 solutions

(r) Does g appear to be even, odd, or neither?
neither

5.1

A Let $f(x) = 2x$ and $g(t) = \dfrac{1}{2t+1}$. Compute the indicated value if it exists.

i. $(f+g)(2)$
$\frac{21}{5}$

ii. $\left(\frac{f}{g}\right)(0)$
0

iii. $(fg)\left(\frac{1}{2}\right)$
$\frac{1}{2}$

B Let f be the function defined by

$$f = \{(-3,4),(-2,2),(-1,0),(0,1),(1,3),(2,4),(3,-1)\}$$

and let g be the function defined by

$$g = \{(-3,-2),(-2,0),(-1,-4),(0,0),(1,-3),(2,1),(3,2)\}$$

Compute the indicated value if it exists.

i. $(g+f)(1)$
0

ii. $\left(\frac{f}{g}\right)(-2)$
does not exist

iii. $(gf)(-3)$
-8

C Let $f(x) = x-1$ and $g(x) = \dfrac{1}{x-1}$, simplify the following expressions.

i. $(f+g)(x)$
$\frac{x^2-2x+2}{x-1}$

ii. $(f-g)(x)$
$\frac{x^2-2x}{x-1}$

iii. $(fg)(x)$
1

iv. $\left(\frac{f}{g}\right)(x)$
x^2-2x+1

D Let $r(x) = \dfrac{3-x}{x+1}$.

i. Find nontrivial[1] functions f and g so that $r = fg$.
Multiple solutions possible, one example: $f(x) = 3 - x$ and $g(x) = \frac{1}{x+1}$.

E Let $f(x) = -3x + 5$.

i. Find and simplify the difference quotient using the formula: $\frac{f(x+h)-f(x)}{h}$
-3

F Let $f(x) = x - x^2$.

i. Find and simplify the difference quotient using the formula: $\frac{f(x+h)-f(x)}{h}$
$-2x - h + 1$

[1]Functions like $f(x) = 1$ do not count.

5.2

A Let $f(x) = 4x + 5$ and $g(t) = \sqrt{t}$, compute the following compositions, if any exist.

i. $(g \circ f)(0)$
$\sqrt{5}$

ii. $(f \circ f)(2)$
57

iii. $(g \circ f)(-3)$
non real answer

B Let f be the function defined by

$$f = \{(-3, 4), (-2, 2), (-1, 0), (0, 1), (1, 3), (2, 4), (3, -1)\}$$

and let g be the function defined by

$$g = \{(-3, -2), (-2, 0), (-1, -4), (0, 0), (1, -3), (2, 1), (3, 2)\}$$

Compute the indicated value if it exists.

i. $(f \circ g)(3)$
4

ii. $(f \circ g)(-3)$
2

iii. $g(f(g(0)))$
-3

iv. $f(f(f(f(f(1)))))$
3

C Let $f(x) = x^2 - x + 1$ and $g(t) = 3t - 5$. Simplify the indicated composition.

i. $(g \circ f)(x)$
$3x^2 - 3x - 2$

ii. $(f \circ g)(t)$
$9t^2 - 33t + 31$

D Let $f(x) = x^2 - x - 1$ and $g(t) = \sqrt{t - 5}$. Simplify the indicated composition.

i. $(g \circ f)(x)$
$\sqrt{x^2 - x - 6}$

ii. $(f \circ g)(t)$
$t - 6 - \sqrt{t - 5}$

E Let $f(x) = -2x$, $g(t) = \sqrt{t}$, and $h(s) = |s|$. Simplify the indicated composition.

i. $(f \circ g \circ h)(s)$
$-2\sqrt{|s|}$

ii. $(h \circ f \circ g)(t)$
$2\sqrt{t}$

iii. $(g \circ h \circ f)(x)$
$\sqrt{2|x|}$

F Write $c(x) = \dfrac{x^2}{x^4 + 1}$ as a composition of two or more non-identity functions.
Let $f(x) = x^2$ and $g(x) = \frac{x}{x^2+1}$, then define $w(x) = (g \circ f)(x)$.

5.3

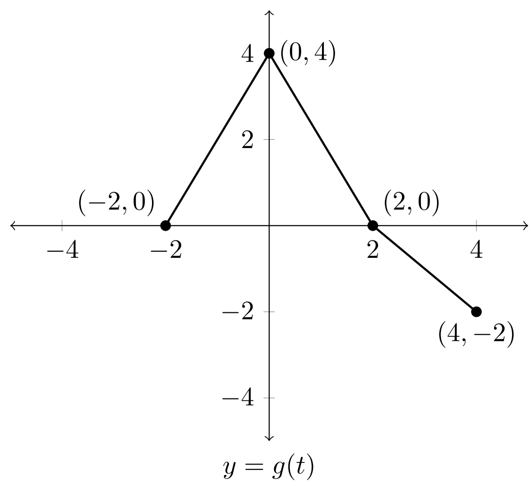
A Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $y = 3f(2x) - 1$.
$(1, -10)$

B Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $y = 5f(2x + 1) + 3$.
$\left(\frac{1}{2}, -12\right)$

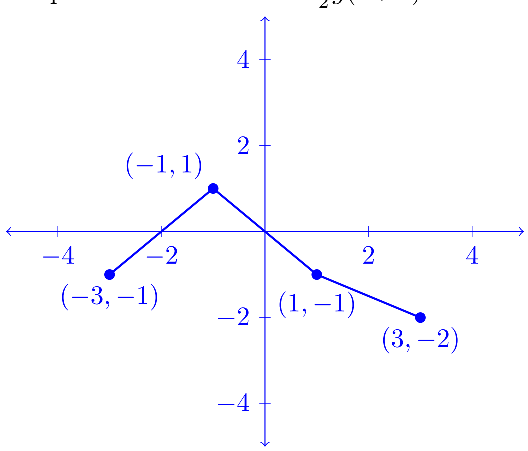
C Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $f\left(\dfrac{7 - 2x}{4}\right)$.
$\left(-\frac{1}{2}, -3\right)$

D Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $\dfrac{4 - f(3x - 1)}{7}$.
$(1, 1)$

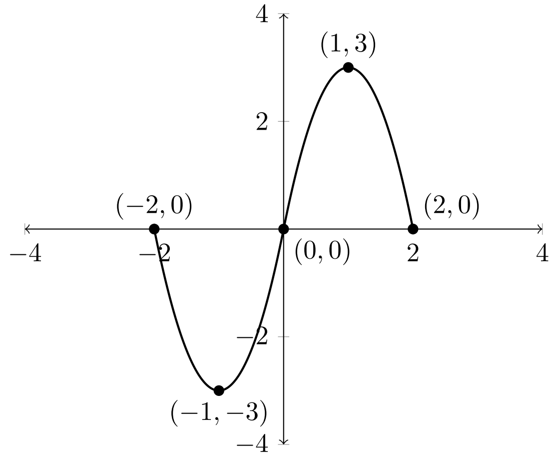
E Given the graph $y = g(t)$



i. Graph the transformation $\frac{1}{2}g(t+1) - 1$

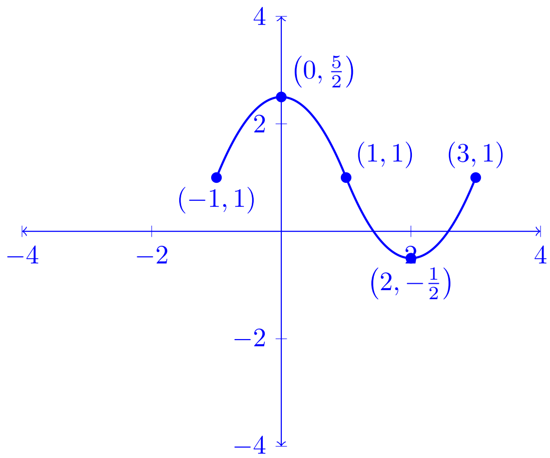


F Given the graph $y = S(t)$



$y = S(t)$

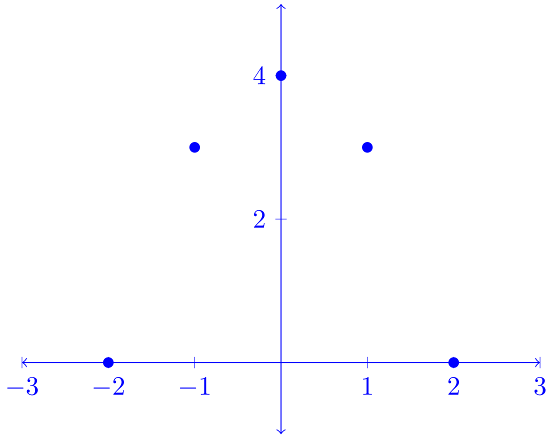
i. Graph the transformation $y = \frac{1}{2}S(-t+1)+1$



5.4

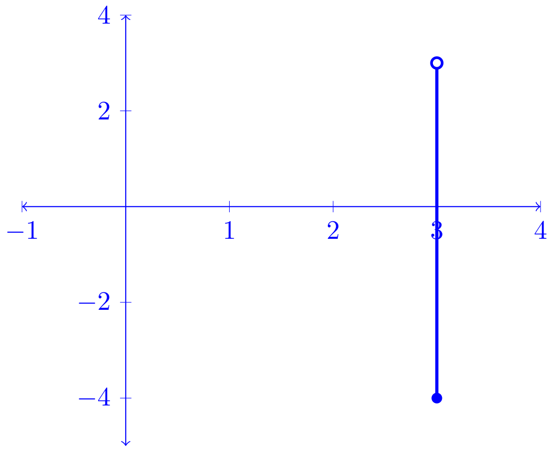
A Graph the indicated relation in the xy-plane.

i. $\{(n, 4 - n^2) \mid n = 0, \pm 1, \pm 2\}$



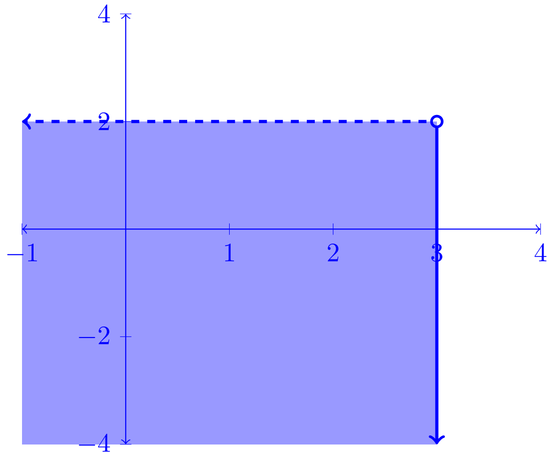
B Graph the indicated relation in the xy-plane.

i. $\{(3, y) \mid -4 \leq y < 3\}$

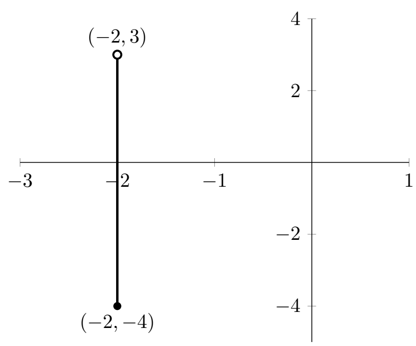


C Graph the indicated relation in the xy-plane.

i. $\{(x, y) \mid x \leq 3, y < 2\}$

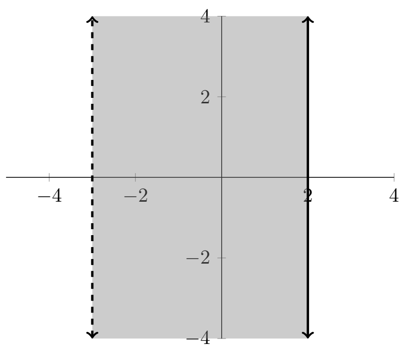


D Describe the given relation using set-builder notation.



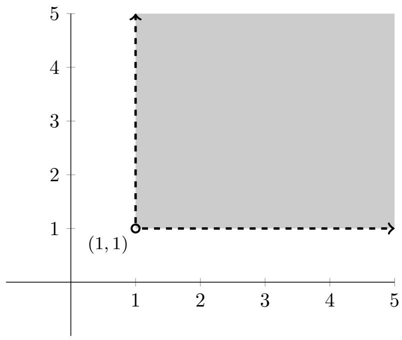
$\{(-2, y) \mid -4 \leq y < 3\}$

E Describe the given relation using set-builder notation.



$\{(x, y) \mid -3 < x \leq 2\}$

F Describe the given relation using set-builder notation.

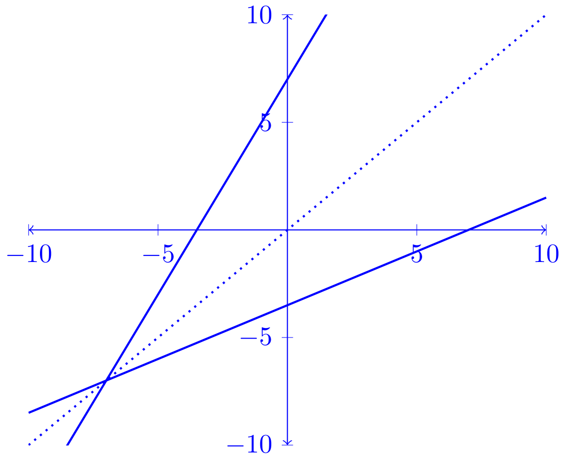


$\{(x, y) \mid 1 < x, 1 < y\}$

5.5

A Let $f(x) = 2x + 7$ and $g(x) = \dfrac{x-7}{2}$.

i. Graph $f(x)$ and $g(x)$ on a coordinate plane.



ii. Are $f(x)$ and $g(x)$ inverse? Justify your answer.
Yes, $f(x)$ and $g(x)$ are reflected over the line $y = x$

B Let $g(t) = \dfrac{t-2}{3} + 4$.

i. Show that $g(t)$ is one-to-one.

ii. Find the inverse of $g(t)$.
$g^{-1}(t) = 3t - 10$

C Let $f(x) = \sqrt{3x-1} + 5$.

i. Show that $f(x)$ is one-to-one.

ii. Find the inverse of $f(t)$.
$f^{-1}(x) = \frac{1}{3}(x-5)^2 + \frac{1}{3}, x \geq 5$

D Let $f(x) = \sqrt[5]{3x-1}$

i. Show that $f(x)$ is one-to-one.

ii. Find $f^{-1}(x)$.
$f^{-1}(x) = \frac{1}{3}x^5 + \frac{1}{3}$

E Let $h(x) = \dfrac{2x-1}{3x+4}$

i. Show that $h(x)$ is one-to-one

ii. Find $h^{-1}(x)$.

$f^{-1}(x) = \dfrac{4x+1}{2-3x}$

F* Under what conditions is $f(x) = mx + b$, $m \neq 0$ its own inverse? Prove your answer.

Proof.

Let $f(x) = mx + b$ with $m \neq 0$. Then $f^{-1}(x) = \frac{x-b}{m} = \frac{1}{m}x - \frac{b}{m}$. For f to be its own inverse we need to verify that $f(x) = f^{-1}(x)$ or in other words that $mx + b = \frac{1}{m}x - \frac{b}{m}$. This yeilds two equations which must both be true.

$$m = \frac{1}{m} \tag{1}$$

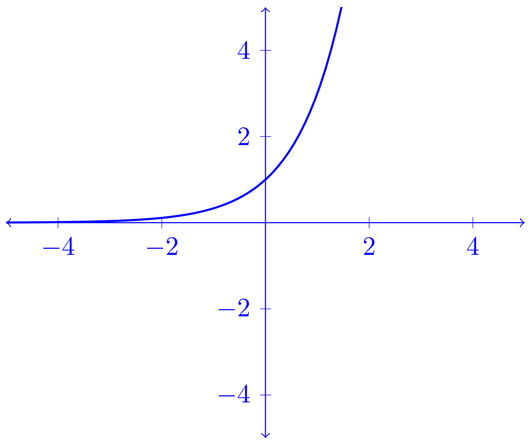
$$b = -\frac{b}{m} \tag{2}$$

Solving for (1) we obtain $m^2 - 1 = 0$ which yields $m = \pm 1$ and solving for (2) we obtain $m = -1$. However the number of solutions for m depends on the value of b. If $b \neq 0$ then m must be -1, however if $b = 0$ either $m = 1$ or $m = -1$ will work. □

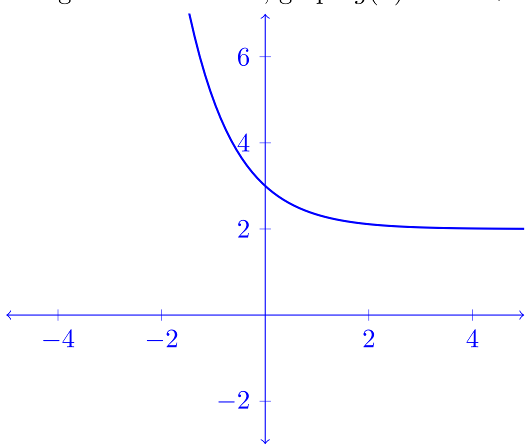
6.1

A Let $f(x) = 3^x$.

i. Sketch the graph of $f(x)$.

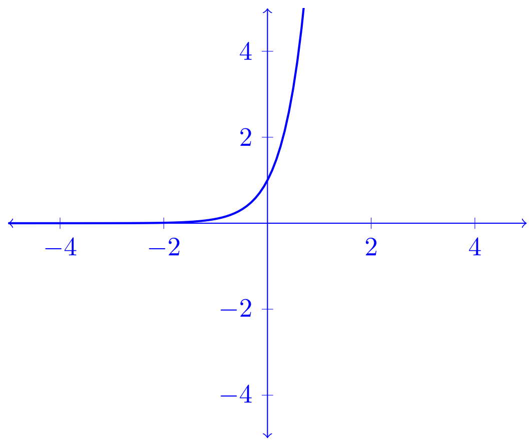


ii. Using transformations, graph $g(x) = 3^{-x} + 2$.

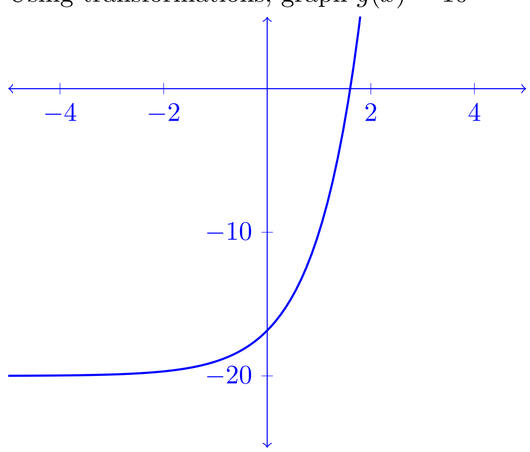


B Let $f(x) = 10^x$

i. Sketch the graph of $f(x)$.

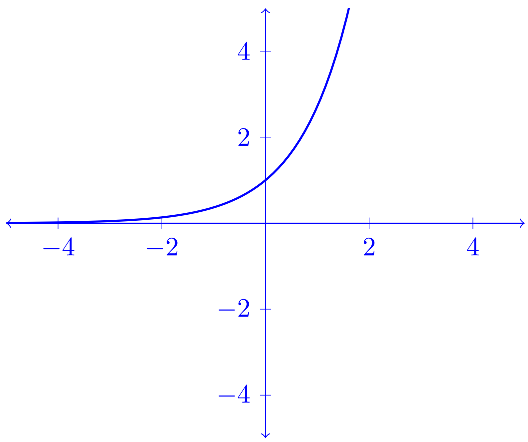


ii. Using transformations, graph $g(x) = 10^{\frac{x+1}{2}} - 20$.

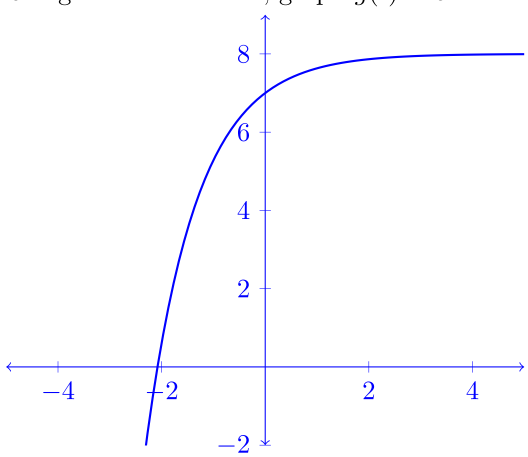


C Let $f(t) = e^t$

i. Sketch the graph of $f(t)$.



ii. Using transformations, graph $g(t) = 8 - e^{-t}$.



D State the domain of $T(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$(-\infty, \infty)$

6.2

A Rewrite the expression: $\log(100) = 2$, so that it does not contain a logarithm.
$100 = 10^2$

B Evaluate $\log_2(32)$.
5

C Evaluate $\log_4(8)$.
$\frac{3}{2}$

D Find the domain of $f(x) = \log_7(t^2 + 9t + 18)$.
$(-\infty, -6) \cup (-3, \infty)$

E Find the domain of $f(x) = \ln(x^2 + 1)$.
$(-\infty, \infty)$

F Find the domain of $g(t) = \ln(7 - t) + \ln(t - 4)$.
$(4, 7)$

6.3

A Expand and simplify: $\ln\left(\frac{\sqrt{z}}{xy}\right)$.
$\frac{1}{2}\ln(z) - \ln(x) - \ln(y)$

B Expand and simplify: $\ln\left(\sqrt[4]{\frac{xy}{ez}}\right)$.
$\frac{1}{4}\ln(x) + \frac{1}{4}\ln(y) - \frac{1}{4} - \frac{1}{4}\ln(z)$

C Write $\frac{1}{2}\log_3(x) - 2\log_3(y) - \log_3(z)$ as a single logarithm.
$\log_3\left(\frac{\sqrt{x}}{y^2 z}\right)$

D Write $\log_5(x) - 3$ as a single logarithm.
$\log_5\left(\frac{x}{125}\right)$

E Write $\log_2(x) + \log_4(x)$ as a single logarithm.
$\log_2(x^{3/2})$

F* With the product rule given, prove the quotient rule and power rule for logarithms.

Proof.

Power Rule: $\log_b(x^y) = \log_b(\underbrace{x \times \cdots \times x}_{y \text{ times}}) = \underbrace{\log_b(x) + \cdots + \log_b(x)}_{y \text{ times}} = y \times \log_b(x)$

Quotient Rule: $\log_b\left(\frac{x}{y}\right) = \log_b\left(x\frac{1}{y}\right) = \log_b\left(xy^{-1}\right) = \log_b(x) + \log_b(y^{-1})$
$= \log_b(x) + (-1 \times \log_b(y)) = \log_b(x) - \log_b(y)$

□

6.4

A Solve $2^{(t^3-t)} = 1$.
$t = \{-1, 0, 1\}$

B Solve $3^{7x} = 81^{4-2x}$.
$x = \frac{16}{15}$

C Solve $e^{2t} = e^t + 6$.
$t = \ln(3)$

D* Solve $7^{3+7x} = 3^{4-2x}$.
$x = \frac{4\ln(3)-3\ln(7)}{7\ln(7)+2\ln(3)}$

E Solve $e^{-x} - xe^{-x} \geq 0$, write your answer in interval notation.
$(-\infty, 1]$

F Solve $(1-e^t)t^{-1} \leq 0$, write your answer in interval notation.
$(-\infty, 0) \cup (0, \infty)$

6.5

A Solve $10\log\left(\frac{x}{10^{-12}}\right) = 150$.
10^3

B Solve $3\ln(t) - 2 = 1 - \ln(t)$.
$t = e^{3/4}$

C Solve $\ln(x+1) - \ln(x) = 3$.
$x = \frac{1}{e^3 - 1}$

D Solve $\ln(t^2) = (\ln(t))^2$.
$t = \{1, e^2\}$

E Solve $\dfrac{1 - \ln(t)}{t^2} < 0$, write your answer in interval notation.
(e, ∞)

F* Solve $\ln(t^2) \leq (\ln(t))^2$, write your answer in interval notation.
$(0, 1] \cup [e^2, \infty)$

7.1

A Convert 135° into radians.

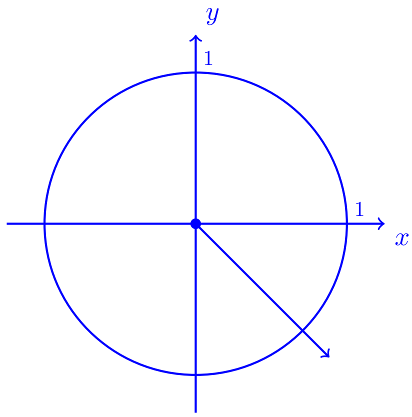
$\frac{3\pi}{4}$

B Convert $\dfrac{5\pi}{3}$ into degrees.

300°

C Let $\theta = \dfrac{15\pi}{4}$

i. Graph θ in standard position.

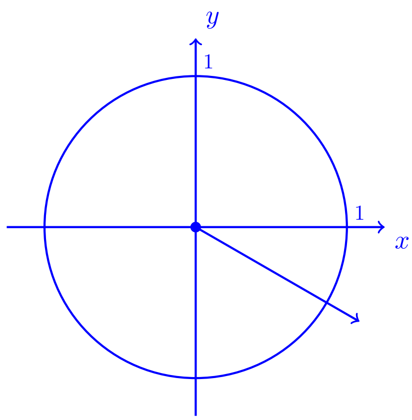


ii. Give two angles coterminal to θ, one which is positive and one which is negative.

More than one answer, one example is: $\frac{7\pi}{4}, -\frac{\pi}{4}$

D Let $\theta = -\dfrac{13\pi}{6}$

i. Graph θ in standard position.



ii. Give two angles coterminal to θ, one which is positive and one which is negative.

More than one answer, one example is: $\frac{11\pi}{6}, -\frac{\pi}{6}$

7.2

A Given $\theta = \dfrac{3\pi}{4}$

i. Find the value of $\sin(\theta)$.
$\frac{\sqrt{2}}{2}$

ii. Find the value of $\cos(\theta)$.
$-\frac{\sqrt{2}}{2}$

B Find all angles which satisfy the equation: $\sin(\theta) = \dfrac{\sqrt{3}}{2}$
$\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{2\pi}{3} + 2\pi k$ where k is any integer.

C Let θ be an angle in standard position whose terminal side contains the point $P(5, -9)$.

i. Compute $\cos(\theta)$.
$\frac{5\sqrt{106}}{106}$

ii. Compute $\sin(\theta)$.
$-\frac{9\sqrt{106}}{106}$

D Assume $\cos(\theta) = -\dfrac{2}{11}$ with θ in Quadrant III.

i. Find the value of $\sin(\theta)$.
$-\frac{\sqrt{117}}{11}$

E Assume $\sin(\theta) = \dfrac{2\sqrt{5}}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$.

i. Find the value of $\cos(\theta)$.
$-\frac{\sqrt{5}}{5}$

F Draw the unit circle from memory.[2]
Google it ☺

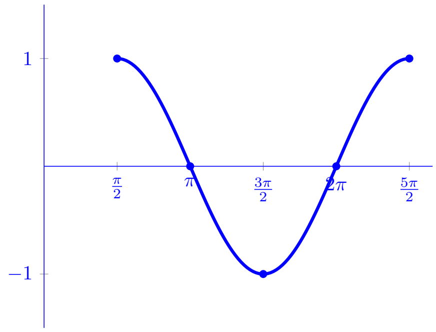
[2] This would not be asked on a test, but you should be able to do this.

7.3

A Let $f(t) = \cos\left(t - \frac{\pi}{2}\right)$

i. State the amplitude, baseline, period, and phase shift of $f(t)$.
Amplitude: 1, Baseline: 0, Period: 2π, Phase Shift: $\frac{\pi}{2}$

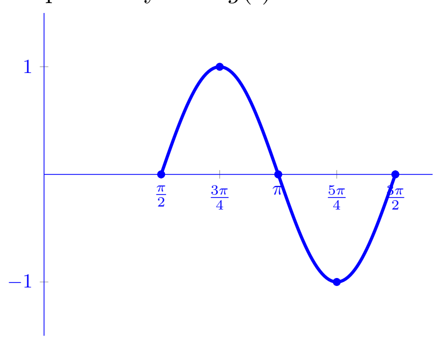
ii. Graph one cycle of $f(t)$.



B Let $g(t) = \sin(2t - \pi)$

i. State the amplitude, baseline, period, and phase shift of $g(t)$.
Amplitude: 1, Baseline: 0, Period: π, Phase Shift: $\frac{\pi}{2}$

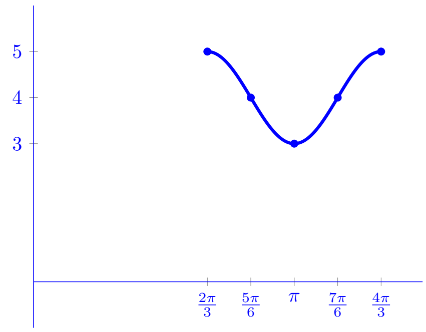
ii. Graph one cycle of $g(t)$.



C Let $h(t) = \cos(3t - 2\pi) + 4$

i. State the amplitude, baseline, period, and phase shift of $h(t)$.
Amplitude: 1, Baseline: 4, Period: $\frac{2\pi}{3}$, Phase Shift: $\frac{2\pi}{3}$

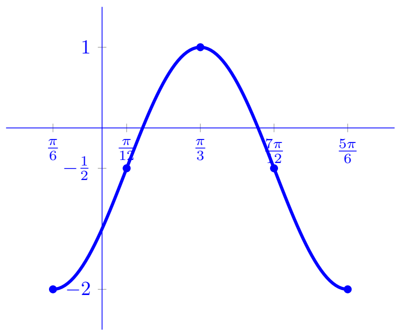
ii. Graph one cycle of $h(t)$.



D Let $q(t) = -\frac{3}{2}\cos\left(2t + \frac{\pi}{3}\right) - \frac{1}{2}$

i. State the amplitude, baseline, period, and phase shift of $q(t)$.
Amplitude: $\frac{3}{2}$, Baseline: $-\frac{1}{2}$, Period: π, Phase Shift: $-\frac{\pi}{6}$

ii. Graph one cycle of $q(t)$.



E* Let S be a collection of sine functions

$$S = \{\sin(\omega_1 x),\, \sin(\omega_2 x),\, \sin(\omega_3 x), \ldots,\, \sin(\omega_n x)\}$$

where no two values of ω are the same. Find a value of x, other than $x = 0$, where all of the sine functions in S equal 0 at the same time.

Proof.

Note that for any arbitrary sine function $\sin(\omega_k x)$ in S, the zeros of the function occur at multiples of $\frac{\pi}{\omega_k}$. So all we need is to find a number that is a multiple of all values of $\frac{\pi}{\omega_k}$. We can use the function $\text{LCM}(a_1, a_2, a_3, \ldots, a_n)$ to denote the least common multiple of n numbers. Define

$$\mathbf{x} = \text{LCM}\left(\frac{\pi}{\omega_1}, \frac{\pi}{\omega_2}, \frac{\pi}{\omega_3}, \ldots, \frac{\pi}{\omega_n}\right)$$

Then $\sin(\omega_1 \mathbf{x}) = \sin(\omega_2 \mathbf{x}) = \sin(\omega_1 \mathbf{x}) = \cdots = \sin(\omega_n \mathbf{x}) = 0$.

□

7.4

A Find the value of $\csc\left(\frac{5\pi}{6}\right)$ if it exists.
2

B Find the value of $\sec\left(-\frac{3\pi}{2}\right)$ if it exists.
undefined

C If it is known that $\sin(\theta) > 0$ but $\tan(\theta) < 0$, in what quadrant does θ lie?
Quadrant II

D Assume $\tan(\theta) = \frac{12}{5}$ with θ in Quadrant III.

i. Find the value of the other five circular functions.
$\sin(\theta) = -\frac{12}{13}, \cos(\theta) = -\frac{5}{13}, \csc(\theta) = -\frac{13}{12}, \sec(\theta) = -\frac{13}{5}, \cot(\theta) = \frac{5}{12}$

E Assume $\cot(\theta) = 2$ with $0 < \theta < \frac{\pi}{2}$

i. Find the value of the other five circular functions.
$\sin(\theta) = \frac{\sqrt{5}}{5}, \cos(\theta) = \frac{2\sqrt{5}}{5}, \tan(\theta) = \frac{1}{2}, \csc(\theta) = \sqrt{5}, \sec(\theta) = \frac{\sqrt{5}}{2}$

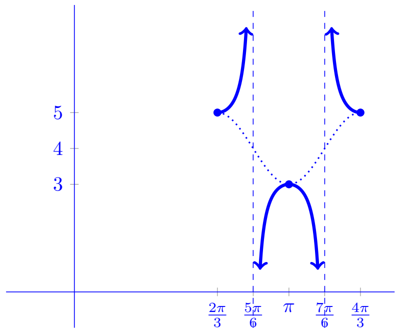
F Find all angles which satisfy the equation $\tan(\theta) = -1$
$\theta = \frac{3\pi}{4} + \pi k$ where k is an integer.

7.5

A Let $f(t) = \sec(3t - 2\pi) + 4$

i. State the period of $f(t)$.
Period: $\frac{2\pi}{3}$

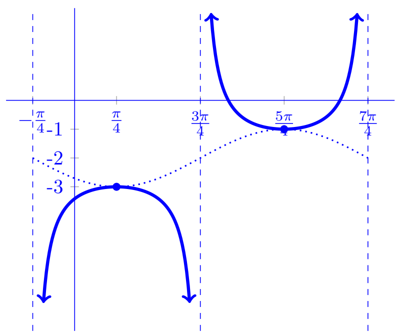
ii. Graph one cycle of $f(t)$.



B Let $g(t) = \csc\left(-t - \frac{\pi}{4}\right) - 2$

i. State the period of $g(t)$.
Period: 2π

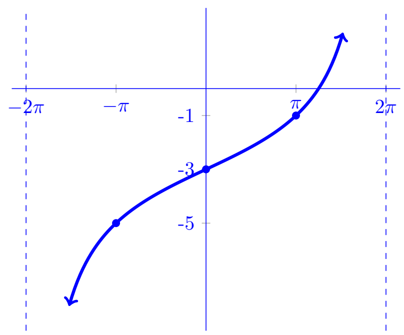
ii. Graph one cycle of $g(t)$.



C Let $r(t) = 2\tan\left(\frac{1}{4}t\right) - 3$

i. State the period of $r(t)$.
Period: 4π

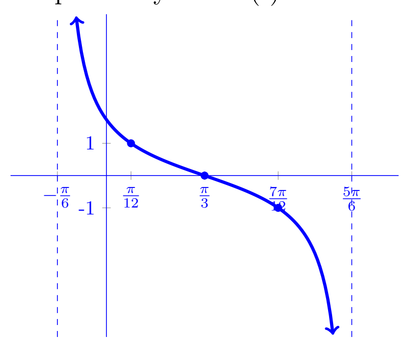
ii. Graph one cycle of $r(t)$.



D Let $s(t) = \cot\left(t + \frac{\pi}{6}\right)$

i. State the period of $s(t)$.
Period: π

ii. Graph one cycle of $s(t)$.



8.1

All identities are true.

A Verify the identity: $\dfrac{\cos(\theta)}{\sin^2(\theta)} = \csc(\theta)\cot(\theta)$

B Verify the identity: $\dfrac{\cos(t)}{1-\sin^2(t)} = \sec(t)$

C Verify the identity: $\tan^3(t) = \tan(t)\sec^2(t) - \tan(t)$

D Verify the identity: $\dfrac{1-\tan(t)}{1+\tan(t)} = \dfrac{\cos(t)-\sin(t)}{\cos(t)+\sin(t)}$

E Verify the identity: $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$

F Verify the identity: $\dfrac{1-\sin(x)}{1+\sin(x)} = (\sec(x)-\tan(x))^2$

8.2

A Find the exact value of $\cos\left(\dfrac{13\pi}{12}\right)$

$-\frac{\sqrt{6}+\sqrt{2}}{4}$

B Find the exact value of $\sin\left(\dfrac{\pi}{12}\right)$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

C Let α be a Quadrant IV angle such that $\cos(\alpha) = \frac{\sqrt{5}}{5}$ and let $\frac{\pi}{2} < \beta < \pi$ such that $\sin(\beta) = \frac{\sqrt{10}}{10}$.

i. Find the value of $\cos(\alpha - \beta)$.
$-\frac{\sqrt{2}}{2}$

D Let $0 < \alpha < \frac{\pi}{2}$ such that $\csc(\alpha) = 3$ and let β be a Quadrant II angle such that $\tan(\beta) = -7$.

i. Find the value of $\tan(\alpha + \beta)$.
$\frac{-28+\sqrt{2}}{4+7\sqrt{2}}$ or reformulated $\frac{63-100\sqrt{2}}{41}$

E Verify the identity: $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos(\alpha)\cos(\beta)$.
True

F Verify the identity: $(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$.
True

8.3

A Find the exact value of $\arccos\left(\frac{1}{2}\right)$

$\frac{\pi}{3}$

B Find the exact value of $\operatorname{arccot}(-1)$

$\frac{3\pi}{4}$

C Find the exact value of $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$

$-\frac{\sqrt{2}}{2}$

D Find the exact value of $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$

$\frac{\sqrt{3}}{2}$

E Solve $\sin(\theta) = \frac{7}{11}$

$\theta = \arcsin\left(\frac{7}{11}\right) + 2\pi k$ or $\theta = \pi - \arcsin\left(\frac{7}{11}\right) + 2\pi k$ where k is an integer.

F State the domain of $\arctan(4x)$

$(-\infty, \infty)$

9.1

Chapter 9 is often not included in a final exam.

A Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 13°$, $\beta = 17°$, and $a = 5$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.
$\gamma = 150°, b \approx 6.50, c \approx 11.11$

B Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 73.2°$, $\beta = 54.1°$, and $a = 117$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.
$\gamma = 52.7°, b \approx 99.00, c \approx 97.22$

C Let (α, a), (β, b), and (γ, c) be angle-side opposite pairs of a triangle such that $\alpha = 95°$, $\beta = 85°$, and $a = 33.33$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.
This information does not produce a triangle.

9.2

Chapter 9 is often not included in a final exam.

A Find the area of the triangle with side lengths, $a = 7$, $b = 10$, and $c = 13$.
$20\sqrt{3}$

B Find the area of the triangle with side lengths, $a = 300$, $b = 302$, and $c = 48$.[3]
$\sqrt{51764375}$

C Find the area of the triangle with side lengths, $a = 5$, $b = 12$, and $c = 13$.
30

[3] Use a calculator.