0.1 | Algebra Review

This sheet contains a list of important topics and properties one should know coming into a Pre-Calculus class. This list is based on problems that show in the Spring 2024 Pre-Test.

Solving Algebraic Equations: The goal of solving an algebraic equation is to identify the variable we are solving for, and isolate it to one side of the equation. We can do this by making any series of mathematical "moves" so long as anything done to one side of the equation is done equally to the other. As long as each move we make is done identically to both sides of the equation, we can continuously reduce the equation until our desired variable is by itself. The side of the equation opposite to our isolated variable will be the solution.

Reciprocals: If we have any fraction $\frac{a}{b}$, the reciprocal is simply the fraction with the numerator and denominator flipped. So the reciprocal of $\frac{a}{b}$ would be $\frac{b}{a}$. We can also take the reciprocal of a number which isn't in the form of a fraction. Simply take the desired number, let's say 13, and write it over top of a 1, and then take the reciprocal. So $\frac{13}{1}$ has a reciprocal of $\frac{1}{13}$.

Zero Product Property: The zero product property states the following. If $a \cdot b = 0$, then either $a = 0$, or $b = 0$ (or both). This might seem trivial, but it is used most often when we have a factored equation set equal to zero. For example, say we have $(x-1)(x+1) = 0$. We can use the zero product property by treating $(x-1)$ as "a", and $(x+1)$ as "b". Then we know that possible solutions involve $(x-1) = 0$, $(x+1) = 0$, or both. We can solve each smaller equation individually to get an overall solution (which in our example is $x = 1$ and $x = -1$).

Exponent Rules: When an equation involves a lot of exponents, we can use a series of rules to simplify the expression. The rules are listed below:

- $a^m \cdot a^n = a^{m+n}$
- $(a^m)^n = a^{m \cdot n}$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a \cdot b)^n = a^n \cdot b^n$
- $a^{-n} = \frac{1}{a^n}$

Arithmetic With Fractions: Fractions can be less intuitive to work with, so I have given reminders of arithmetic rules here:

- **Addition**: When adding two fractions, find a common denominator, then add across the numerators. For example: $\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}$
- **Subtraction**: When subtracting two fractions, find a common denominator, then subtract across the numerators. For example: $\frac{a}{d} - \frac{b}{d} = \frac{a-b}{d}$
- **Multiplication**: When multiplying fractions, you can simply multiply through the numerator and denominator no matter what the values are. For example: $\frac{a}{c} \cdot \frac{b}{d} = \frac{a \cdot b}{c \cdot d}$

- **Division**: When dividing fractions, you can equivalently multiply by the reciprocal. For example: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$

Changing Denominators: Changing a denominator or finding a common denominators for fractions can be very useful for adding or subtracting, as well as comparing which fractions are larger than others. To change a fraction's denominator, we want to multiply the fraction by something that equals 1 (because multiplication by 1 doesn't change the value of something). A fraction that equals one is simply the same thing over itself, so any fraction that looks like $\frac{a}{a}$ is equal to 1. We can choose any value for a that we want, and then multiply it by the desired fraction to change the denominator.

Midpoint and Averaging: If we are given two values and asked to find a point that lies between them, we can average the two values. Given values a and b, we can add them together and divide by 2 to find the exact center point of the two values: $\frac{a+b}{2}$. If we want to do the same thing but with two points on a graph, let's say (x_1, y_1) and (x_2, y_2), we can use this formula where M is the coordinate pair of the midpoint: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

List of Squares: It is good to be familiar with the square of the numbers 1 through 12, they are listed here.

- $1^2 = 1$
- $2^2 = 4$
- $3^2 = 9$
- $4^2 = 16$
- $5^2 = 25$
- $6^2 = 36$
- $7^2 = 49$
- $8^2 = 64$
- $9^2 = 81$
- $10^2 = 100$
- $11^2 = 121$
- $12^2 = 144$

Dividing by Zero: Dividing by zero gives an *undefined* value. We can also write 'DNE' to stand for "Does Not Exist" if the division by zero is the solution to a problem.

Pythagorean Theorem: Recall that given a right triangle (one angle is 90°) where the legs are a and b and the hypotenuse (the longest side) is c, the following relationship holds: $a^2 + b^2 = c^2$.

Multiplying Square Roots: The following relationship holds only when a and b are **both positive:** $\sqrt{a \cdot b} = \sqrt{a}\sqrt{b}$.