3.3 | Inequalities involving Rational Functions

Note: This section is titled "Inequalities involving Rational Functions" but this is a slight lie. We will also be covering equations involving rational functions.

Solving Equations involving Rational Functions: The general process for solving equations with rational functions goes as follows.

- Move all terms to one side of the equation (so the other side is 0).
- Put all terms under a **common denominator** (so the entire equation is one fraction).
- **Factor** the numerator and denominator, cancel terms where you can.
- Set the numerator equal to zero and solve. These will be the **zeros of the rational function**.
- Check your solutions.

It is important to check your solutions here, as sometimes in the process of moving equations around and finding common denominators we can accidentally solve for a zero that was an asymptote in the original problem. You want to verify that any zeros you find are included in the domain of the function.

1. **Worked Example:** Solve the rational equation: $\dfrac{x}{5x+4} = 3$

Scan the QR code for a video solution.

2. Solve the rational equation: $\dfrac{3x-1}{x^2+1} = 1$

3. Solve the rational equation: $\dfrac{2t+17}{t+1} = t+5$

4. Solve the rational equation: $\dfrac{1}{t+3} + \dfrac{1}{t-3} = \dfrac{t^2-3}{t^2-9}$

Solving inequalities for rational functions follows a similar process as solving inequalities of polynomial functions. We may need to do a little bit of extra work to make sure the equation is in the form we want and then we simply adjust our sign diagram method as outlined in section 3.2.

Solving Inequalities for Rational Functions:

- Move all terms to one side of the inequality (so the other side is 0).
- Put all terms under a **common denominator** (so the entire equation is one fraction).
- **Factor** the numerator and denominator, cancel terms where you can.
- Set the numerator equal to zero and solve, place these points as **zeros on your sign diagram**.
- Set the denominator equal to zero and solve, place these points as **vertical asymptotes on your sign diagram**.
- Compute then interpret the sign diagram.

Points on the sign diagram that are labeled as vertical asymptotes will never be included in the solution interval, regardless of the inequality symbol used in the equation. Points labeled as zero will be included or not included depending on the $\leq$ or $<$ symbol respectively.

5. **Worked Example:** Solve the rational inequality: $\dfrac{5}{x+2} \geq 1$

Scan the QR code for a video solution.

6. Solve the rational inequality: $\dfrac{1}{x+2} \geq 0$

7. Solve the rational inequality: $\dfrac{x}{x^2-1} < 0$

8. Solve the rational inequality: $\dfrac{4t}{t^2+4} \geq 0$

9. Solve the rational inequality: $\dfrac{2t+6}{t^2+t-6} < 1$

10. Solve the rational inequality: $\dfrac{6}{z-1} + 1 > \dfrac{1}{z+1}$

11. Solve the rational inequality: $\left(x^2+1\right)^{-1} < 0$