6.5 | Equations and Inequalities involving Logarithmic Functions

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Solving Logarithmic Equations: If we want to solve a logarithmic equation, we can generally follow these steps. Each equation is different, however, so adjustments may need to be made. The best way to improve at solving equations is to practice.

- Isolate the logarithm.
- Take the appropriate exponential of both sides to reduce the equation. Repeat if necessary.
- Solve the equation like a polynomial.
- Check solutions (keep in mind $f(x) = \log_b(x)$ cannot take a negative input).

> **Problem Solving Tip 1.** If you want to reduce a logarithm of base b, you will need to take the exponential of base b of both sides. This is an extremely important idea that many students forget to utilize. This is an application of the following property: $b^{\log_b(x)} = x$.

Note About Solutions: In this section, some final solutions may look different than what we are used to. For example, the solution $x = e^3$ is perfectly valid. To humans, this may look arbitrary, and kind of weird, but we leave it in this form as a decimal approximation would require an infinite amount of digits.[1] When a solution contains natural numbers instead of e, it is often easiest to compute powers for lower values. In a special case where the power is too large to compute[2], leaving the solution in exponential form is acceptable.

1. **Worked Example:** Solve: $\log(3x - 1) = \log(4 - x)$

Scan the QR code for a video solution.

[1] For those curious, $e^3 \approx 20.0855369231876677409285296545817178969879078385541501443789342 29\ldots$

[2] Don't get lazy with this however. Something like 2^{11} is actually fairly reasonable to compute. I am talking about solutions like 8^6, where the solution 262144 could not be computed in a reasonable time frame.

2. Solve: $\log_2(x^3) = \log_2(x)$

3. Solve: $\ln(8 - t^2) = \ln(2 - t)$

4. Solve: $\log_3(7 - 2x) = 2$

5. Solve: $\ln(t^2 - 99) = 0$

6. Solve: $\log_{125}\left(\frac{3x-2}{2x+3}\right) = \frac{1}{3}$

7. Solve: $10\log\left(\frac{x}{10^{-12}}\right) = 150$

8. Solve: $-\log(x) = 5.4$

9. Solve: $3\ln(t) - 2 = 1 - \ln(t)$

10. Solve: $\log_3(t-4) + \log_3(t+4) = 2$

11. Solve: $\ln(x+1) - \ln(x) = 3$

12. Solve: $\ln(\ln(x)) = 3$

13. **Challenge Problem:** Solve: $\ln(t^2) = (\ln(t))^2$

Scan the QR code for a worked solution

Solving Logarithmic Inequalities: When trying to solve an inequality, again every problem is different. However here are some common methods we can try:

- Reduce the logarithms then solve like a polynomial.
- Factor the expression and make a sign diagram. Keep in mind $f(x) = \log_b(x)$ is negative when $x < 1$ and positive when $x > 1$.

14. Solve the inequality: $\dfrac{1 - \ln(t)}{t^2} < 0$

15. Solve the inequality: $t\ln(t) - t > 0$

16. Solve the inequality: $10\log\left(\dfrac{x}{10^{-12}}\right) \geq 90$

17. Solve the inequality: $2.3 < -\log(x) < 5.4$

18. **Challenge Problem:** $\ln(t^2) \leq (\ln(t))^2$

Scan the QR code for a worked solution