MATH1300
Selected Challenge Problems
Volume II

Precalculus Peer Assisted Learning

March 4, 2025

Preface:

These problems are a compilation of problems from the textbook, along with some of my own creations, with the intent to provide a decent challenge to anyone in MATH1300. My goal is that if one is able to *confidently* complete the majority of problems in each section, they should be fairly well prepared for the exam. *Generally* speaking, the problems become more difficult as you move from **A** to **F**, although some students may find earlier problems more difficult and later problems easier. If you find yourself struggling to start *any* problem at all, you may want to go back and review easier questions from the book or my other worksheets before returning to these problems.

Problems with an **asterisk*** (usually problem **F**) are exceptionally difficult and require a deeper level of introspection into the topic to solve. I would suggest attempting problems with an asterisk only after all preceding problems have been successfully completed. Problems with an asterisk are likely much more difficult than what would show up on an exam, don't worry if you are unable to solve them.

Keep in mind, I have no special insider information on what will actually appear on the exam, and **you should not take this booklet as a representation or study guide of what you will see on your exam.** If you plan to do well on the exam, you **MUST** spend time studying content and problems that appear outside of this booklet.

Roman

2.2

A Let $f(z) = 4z^3 + 2z - 3$ and $g(z) = z - 3$

i. Compute $f(z)/g(z)$.

ii. Write $f(z)$ as an expression involving $g(z)$, a quotient, and remainder (if it exists).

B Let $f(x) = 2x^3 - x + 1$ and $g(x) = x^2 + x + 1$

i. Compute $f(x)/g(x)$.

ii. Write $f(x)$ as an expression involving $g(x)$, a quotient, and remainder (if it exists).

C Let $a(x) = x^4 - 6x^2 + 9$ and $b(x) = (x - \sqrt{3})$

i. Compute $a(x)/b(x)$.

ii. Write $a(x)$ as an expression involving $b(x)$, a quotient, and remainder (if it exists).

D Let $g(z) = z^3 + 2z^2 - 3z - 6$ be a polynomial function with a known real zero of $c = -2$

i. Find the remaining real zeros of $g(z)$

E Let $x^3 - 6x^2 + 11x - 6$ be a polynomial function with a known real zero of $c = 1$

i. Find the remaining real zeros of $g(z)$

F* Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial function with the property that $a_n + a_{n-1} + \cdots + a_1 + a_0 = 0$. (That is, the sum of the coefficents and the constant term is 0.)

i. Show that $(x - 1)$ is a factor of $f(x)$.

2.3

A Let $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$

i. Use Cauchy's Bound to find an interval containing all possible rational zeros.

ii. Use the Rational Zeros Theorem to make a list of possible rational zeros.

iii. Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros.

B Let $p(z) = 2z^4 + z^3 - 7z^2 - 3z + 3$

i. Use the Rational Zeros Theorem to list possible roots of the polynomial.

ii. Write the polynomial in factored form.

C Let $g(x) = x^4 - 9x^2 - 4x + 12$

i. Sketch the graph of $g(x)$.

ii. Label all axis intercepts on the graph.

iii. Write the end behavior of $g(x)$.

D Solve the following equation: $x^3 + x^2 = \dfrac{11x + 10}{3}$

E Let $f(x) = (x - 1)^2$ and $g(x) = 4$

i. Graph $f(x)$ and $g(x)$ on the same coordinate plane.

ii. Solve the inequality $f(x) \geq g(x)$ graphically.

iii. Solve the inequality $f(x) \geq g(x)$ algebraically and verify that it matches the solution found in part (ii).

F Solve the inequality: $\dfrac{x^3 + 20x}{8} \geq x^2 + 2$, express your answer in interval notation.

3.1

A Let $p(x) = 9x^3 + 5$ and $q(x) = 2x - 3$

i. Divide $p(x) \div q(x)$ using synthetic division or long division.

ii. Write $p(x)$ in the form of $d(x)q(x) + r(x)$.

B Let $p(x) = 4x^2 - x - 23$ and $q(x) = x^2 - 1$

i. Divide $p(x) \div q(x)$ using synthetic division or long division.

ii. Write $p(x)$ in the form of $d(x)q(x) + r(x)$.

C Let $h(x) = \dfrac{2x}{x+1}$.

i. Write $h(x)$ in the form $\dfrac{a}{x-h} + k$.

ii. Write the parent function $P(x)$ of $h(x)$.

iii. Track at least two points and the asymptotes from $P(x)$ and use them to graph $h(x)$.

D Let $r(x) = \dfrac{x^2 - x - 12}{x^2 + x - 6}$

i. Identify any holes in the graph of $r(x)$.

ii. Identify any vertical asymptotes in the graph of $r(x)$.

iii. State the domain of $r(x)$.

E Let $f(x) = \dfrac{x^3 + 2x^2 + x}{x^2 - x - 2}$

i. Identify any holes in the graph of $f(x)$.

ii. Identify any vertical asymptotes in the graph of $f(x)$.

iii. State the domain of $f(x)$.

F* Let $u(x)$ be a function defined only on the positive real numbers. Let $v(x) = (x - a)(x + b)$ with $0 < a < b$.

i. State the domain of $w(x) = \dfrac{u(x)}{v(x)}$

3.2

A Let $f(x) = 5x(6 - 2x)^{-1}$

i. Sketch the graph of $f(x)$. Label all asymptotes, holes, and zeros.

B Let $a(x) = \dfrac{x}{x^2 + x - 12}$

i. Sketch the graph of $a(x)$. Label all asymptotes, holes, and zeros.

C Let $r(t) = \dfrac{t^2 - t - 6}{t + 1}$

i. Sketch the graph of $r(t)$. Label all asymptotes, holes, and zeros.

D Let $f(x) = \dfrac{5x}{9 - x^2} - x$

i. Sketch the graph of $f(x)$. Label all asymptotes, holes, and zeros.

E Let $r(z) = -z - 2 + \dfrac{6}{3 - z}$

i. Sketch the graph of $r(z)$. Label all asymptotes, holes, and zeros.

F* Let $p(x) = 2x^3 + 5x^2 + 4x + 3$ and $q(x) = 2x + 1$

i. Does $r(x) = \dfrac{p(x)}{q(x)}$ have a horizontal or slant asymptote?

ii. Divide $p(x) \div q(x)$ and ignore the remainder. What does this suggest about the (non vertical) asymptote of $r(x)$?

iii. Assume $a(x)$ is a fourth degree polynomial, and $b(x)$ is a linear. Assuming $b(x)$ is not a factor of $a(x)$, what might the (non vertical) asymptote of $f(x) = \dfrac{a(x)}{b(x)}$ look like?

3.3

A Solve $\dfrac{3x-1}{x^2+1} = 1$.

B Solve $\dfrac{1}{t+3} + \dfrac{1}{t-3} = \dfrac{t^2-3}{t^2-9}$.

C Solve $\dfrac{4t}{t^2+4} \geq 0$.

D Solve $\dfrac{2t+6}{t^2+t-6} < 1$.

E Solve $\dfrac{3z-1}{z^2+1} \leq 1$.

F* Solve $\dfrac{2x^2-5x+4}{3x^2+1} < 0$, justify your answer.

4.1

A Let $f(x) = \sqrt{4-x} - 1$

i. Write the parent function $P(x)$ for f.

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

B Let $f(x) = -\sqrt[3]{8x+8} + 4$

i. Write the parent function $P(x)$ for f.

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

C Let $f(x) = -3\sqrt[4]{x-7} + 1$

i. Write the parent function $P(x)$ for f.

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

D Let $d(x) = \dfrac{5x}{\sqrt[3]{x^3+8}}$

i. State the domain of $d(x)$.

E Let $z(x) = \sqrt{x(x+5)(x-4)}$

i. State the domain of $z(x)$.

F Let $c(x) = \sqrt[6]{\dfrac{x^2+x-6}{x^2-2x-15}}$

i. State the domain of $c(x)$.

4.2

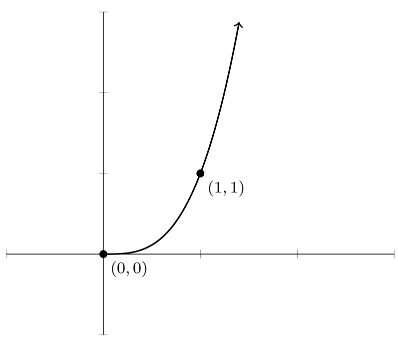
A Let $c(x) = x^{\frac{4}{7}}$

i. List the intervals where $c(x)$ is increasing (if any exist).
ii. List the intervals where $c(x)$ is decreasing (if any exist).
iii. List the intervals where $c(x)$ is concave up (if any exist).
iv. List the intervals where $c(x)$ is concave down (if any exist).

B Let $b(t) = t^{\frac{10}{4}}$

i. List the intervals where $c(x)$ is increasing (if any exist).
ii. List the intervals where $c(x)$ is decreasing (if any exist).
iii. List the intervals where $c(x)$ is concave up (if any exist).
iv. List the intervals where $c(x)$ is concave down (if any exist).

C The graph $g(t) = t^{\pi}$ is shown (where $\pi \approx 3.1415\ldots$).



i. Track the points provided on $g(t)$ to graph $G(t) = \left(\frac{t+3}{2}\right)^{\pi} - 1$

D Let $f(x) = x^{\frac{3}{2}}(x-7)^{\frac{1}{3}}$

i. State the domain of $f(x)$.

E Let $g(t) = t^{\frac{3}{2}}(t-2)^{-\frac{1}{2}}$

i. State the domain of $f(x)$.

F* Let $g(t) = 4t(9-t^2)^{-\sqrt{2}}$

i. State the domain of $g(t)$.

4.3

A Solve the equation $2x + 1 = (3 - 3x)^{\frac{1}{2}}$

B Solve the equation $(2x + 1)^{\frac{1}{2}} = 3 + (4 - x)^{\frac{1}{2}}$

C Solve the equation $2t^{\frac{1}{3}} = 1 - 3t^{\frac{2}{3}}$

D Solve the inequality $\sqrt[3]{x} > x$, express your answer in interval notation.

E Solve the inequality $(2 - 3x)^{\frac{1}{3}} > 3x$, express your answer in interval notation.

F Solve the inequality $3(x - 1)^{\frac{1}{3}} + x(x - 1)^{-\frac{2}{3}} \geq 0$, express your answer in interval notation.