# MATH1300
## Selected Challenge Problems
*Complete Edition*

Precalculus Peer Assisted Learning

Spring 2025

# Contents

## Resources

*You may find the following online resources helpful in your studies.*

### WolframAlpha

Wolfram Alpha is a website that is capable of solving many mathematical equations, it a great resource for checking answers, and speeding up lengthy computations. The website often gives more advanced answers than the user requests, so it may take some time to navigate and find the features you are looking for.

`wolframalpha.com`

### Desmos

Desmos is a online graphing calculator that is capable of graphing many different types of equations. It is a powerful resource for checking if an equation you sketched is accurate, and modeling different equations and systems.

`desmos.com/calculator`

### The Organic Chemistry Tutor

Despite the misleading name, the YouTube channel The Organic Chemistry Tutor makes a large variety of STEM videos that help many students in almost every undergraduate subject. If you somehow aren't already aware of this channel, the QR code will take you to his precalculus playlist.

`youtube.com/@TheOrganicChemistryTutor`

### Professor Leonard

Professor Leonard is a lesser known math YouTuber who uploads fantastic videos on a variety of subjects. He has a very easy to understand teaching style and takes time to explain all steps. His precalculus playlist can be found in the QR code.

`youtube.com/@ProfessorLeonard`

# Study Tips

*For many readers, this is the first college math final you will have taken, but the increase in difficulty is nothing to be afraid of. Disciplined and routine study sessions are the key to success. Feel free to incorporate these study tips in your preparation.*

- **Start early**
  While this is perhaps the most self explanatory, it is the most important. It is very important that you are you able to take in information, and the sleep on it (literally). If you try to cram every ounce of studying in the night before, not only will you not remember as much, but your energy levels for the exam will be completely depleted.

- **Make a study group**
  Studying with a group of people can help you to gain insight into problems that you previously were having trouble solving. You also tend to pickup on tips and tricks along the way when watching others solve problems. Just make sure the group you choose is dedicated to actually studying. If your study group is distracting you, it can end up doing more harm than good.

- **Solve problems without notes**
  When studying, it can be temping to check your notes the minute you get confused, but keep in mind that during the final, you will not have any help. When you get stuck, do you best to reason through the problem as much as possible, and make as much progress without any external help. Once you have done everything you can, check your notes and look back at your work as if you were critiquing another person. It can help give you insight into your specific strengths and weaknesses when taking an exam.

- **Take breaks**
  The first tip helps make this more possible, but taking frequent and meaningful breaks can help increase your productivity. A common method is that every 30 minutes you spend studying, you should take a 5 minute break. It is helpful to walk away from your desk during this break and remove yourself from the mindset of studying so you can be in a fresh mind state before returning.

- **Ask for help**
  Some students feel awkward approaching others for help on a math problem. Do not do this. Take advantage of your resources around you. If you are genuinely stuck on a problem or confused by a topic, find a professor, PAL leader, tutor, or classmate who can help you.

- **Relax the night before the test**
  The night before the test (or the morning of), it might not be the best idea to cram study. You likely already know everything you can, and forcing yourself to study further will only drain your energy. Relax, watch a show you like, or enjoy some time outside. Allow yourself to be in a clear head space so that you can conserve energy needed to take a final exam.

## *Preface:*

These problems are a compilation of problems from the textbook, along with some of my own creations, designed to form multi-step problems that provide a decent challenge to anyone in MATH1300. My goal is that if you are able to solve the majority of the problems in each section, then you have demonstrated a thorough understand of the topics throughout the course. *Generally* speaking, the problems become more difficult as you move from **A** to **F**, although some students may find earlier problems more difficult and later problems easier. If you find yourself struggling to start *any* problem at all, you should spend more time reviewing the topics themselves before attempting these questions. Problems with an **asterisk*** are exceptionally difficult and require a deeper level of introspection into the topic to solve. If you are able to solve a problem with an asterisk, you likely have enough knowledge of the given section to perform well on the exam (no promises).

Keep in mind, I have no special insider information on what will actually appear on the exam, and **you should not take this booklet as a representation of what you will see on your exam.**

## *Note from author:*

To all my PAL students who show up the sessions, exam reviews, and send me emails: thank you. I love this job, and you all contribute to making it something special for me. Best of luck on this final, and in your future studies.

–*Roman*

## 1.1

**A** Observe the following equation: $2xy = 4$.

i. Does this equation represent $y$ as a function of $x$?

ii. If so, write the domain of the equation as set, if not, provide an example where it fails as a function.

**B** Observe the set of ordered pairs

$$\{(-3, 9), (1, 1), (3, 1), (0, 0), (-2, 4), (-3, 7), (4, 0)\}$$

i. Does the set of ordered pairs represent a function?

ii. If so, write the domain as a set, if not, provide an example where it fails as a function.

**C** Observe the following data table.

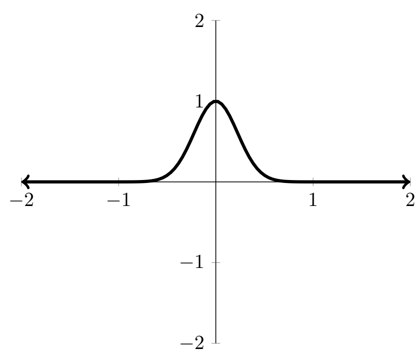
| $x$ | $y$ |
|---|---|
| $-3$ | 3 |
| $-2$ | 2 |
| $-1$ | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

i. Does the given table represent $y$ as a function of $x$? Explain.

ii. Write the domain of the table as a set.

iii. Write the range of the table as a set.

**D** Observe the graph

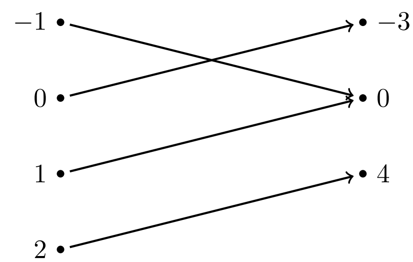



i. Does the graph represent a function? Explain.

ii. Write the domain of the graph using interval notation.

iii. Write the range of the graph using interval notation.

**E** Consider the function $f$ as a mapping diagram shown:




i. Write the domain of $f$ as a set.
ii. Write the range of $f$ as a set.
iii. Find $f(0)$ and solve $f(x) = 0$.
iv. Write $f$ as a set of ordered pairs.

**F** Let $g = \{(-1, 4), (0, 2), (2, 3), (3, 4)\}$

i. Write the domain of $g$ as a set.
ii. Write the range of $g$ as a set.
iii. Find $g(0)$ and solve $g(x) = 0$.
iv. Create a mapping diagram for $g$.

## 1.2

**A** Graph the function $h(t) = \frac{2}{3}t + \frac{1}{3}$.

i. What is the slope?
ii. State the axis intercepts, if they exist.

**B** Graph the function $j(w) = \dfrac{1-w}{2}$

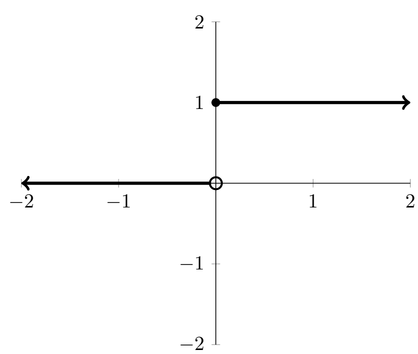
i. What is the slope?
ii. State the axis intercepts, if they exist.

**C** Find the equation of the function that contains the points $(1, 3)$ and $(3, 2)$.

**D** Graph the piecewise function $f(x) = \begin{cases} 4 - x & \text{if } x \leq 3 \\ 2 & \text{if } x > 3 \end{cases}$

i. Write the domain in interval notation.
ii. Write the range in interval notation.
iii. State the axis intercepts, if they exist.

**E** The unit step function is graphed below:




i. Write the equation $U(t)$ of the unit step function.
ii. Write the domain of $U(t)$
iii. Write the range of $U(t)$

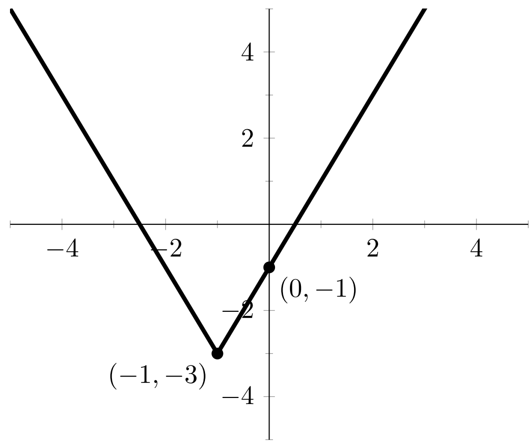
**F*** Explain why the graph of a function $f(x)$ must have at most one $y$-intercept.

## 1.3

**A** Graph the function $g(t) = 3|t+4| - 4$

i. Write the domain of $g(t)$ in interval notation.
ii. Write the range of $g(t)$ in interval notation.
iii. State the axis intercepts, if they exist.

**B** The graph of $F(x)$ is shown below:




i. Write piecewise function definition of $F(x)$.
ii. State the domain of $F(x)$.
iii. State the range of $F(x)$.

**C** Graph the function $g(x) = |t+4| + |t-2|$.

i. Write the domain of $g(x)$ using interval notation.
ii. Write the range of $g(x)$ using interval notation.
iii. State axis intercepts, if they exist.

**D** Solve the equation $|3x-2| = |2x+7|$.

i. Write the solutions as a set.

**E** Given $f(x) = |3x-5|$ and $g(x) = 4$

i. Graph $f(x)$.
ii. Graph $g(x)$ (on the same plot).
iii. Solve $f(x) \leq g(x)$. Write your answer in interval notation.

**F*** Show that if $d$ is a real number with $d > 0$, the solution to $|x-a| < d$ is the interval: $(a-d, a+d)$. That is, an interval centered at $a$ with 'radius' $d$.

## 1.4

**A** Let $f(x) = x^2 - 2x - 8$

i. Complete the square on $f(x)$.

ii. Write the vertex.

iii. Find the axis intercepts.

iv. Graph $f(x)$.

**B** Let $h(t) = -3t^2 + 5t + 4$

i. Compute the discriminant of $h(t)$. How many real zeros does $h(t)$ have?

ii. Find the zero(s) of $h(t)$ if they exist, write your solutions as a set.

**C** Let $g(x) = x^2 - 3x + 9$

i. Is $g(x)$ factorable?

ii. If yes, write $g(x)$ in factored form. If not, explain why.

**D** Solve the inequality $3x^2 \leq 11x + 4$, write your answer in interval notation.

**E** Solve the inequality $5t + 4 \leq 3t^3$, write your answer in interval notation.

**F*** Graph $f(x) = |1 - x^2|$.

## 2.1

**A** Let $g(x) = 3x^5 - 2x^2 + x + 1$

i. Identify the degree of $g(x)$.
ii. Identify the leading coefficient of $g(x)$.
iii. Identify the leading term of $g(x)$.
iv. Identify the constant term of $g(x)$.
v. Write the end behavior of $g(x)$.

**B** Let $f(x) = 3(x + 2)^3 + 1$

i. Write the parent function $P(x)$ for $f(x)$.
ii. Pick three points from the parent function $P(x)$ and apply the transformations of $f(x)$ to write three points on the graph of $f(x)$.
iii. Sketch the graph of $f(x)$.
iv. State the domain and range of $f(x)$ using interval notation.

**C** Let $f(x) = 2 - 3(x - 1)^4$

i. Write the parent function $P(x)$ for $f(x)$.
ii. Pick three points from the parent function $P(x)$ and apply the transformations of $f(x)$ to write three points on the graph of $f(x)$.
iii. Sketch the graph of $f(x)$.
iv. State the domain and range of $f(x)$ using interval notation.

**D** Let $h(t) = t^2(t - 2)^2(t + 2)^2$

i. List all zeros of $h(t)$ and their corresponding multiplicities.
ii. Write the end behavior of $h(t)$.
iii. Sketch a graph of the function $h(t)$.

**E** Let $g(x) = (2x + 1)^2(x - 3)$

i. List all zeros of $g(x)$ and their corresponding multiplicities.
ii. Write the end behavior of $g(x)$.
iii. Sketch a graph of the function $g(x)$.

**F** Let $f(x) = (x^2 + 1)(x - 1)$

i. Determine analytically if $f(x)$ is even, odd, or neither.

## 2.2

**A** Let $f(z) = 4z^3 + 2z - 3$ and $g(z) = z - 3$

i. Compute $f(z)/g(z)$.

ii. Write $f(z)$ as an expression involving $g(z)$, a quotient, and remainder (if it exists).

**B** Let $f(x) = 2x^3 - x + 1$ and $g(x) = x^2 + x + 1$

i. Compute $f(x)/g(x)$.

ii. Write $f(x)$ as an expression involving $g(x)$, a quotient, and remainder (if it exists).

**C** Let $a(x) = x^4 - 6x^2 + 9$ and $b(x) = (x - \sqrt{3})$

i. Compute $a(x)/b(x)$.

ii. Write $a(x)$ as an expression involving $b(x)$, a quotient, and remainder (if it exists).

**D** Let $g(z) = z^3 + 2z^2 - 3z - 6$ be a polynomial function with a known real zero of $c = -2$

i. Find the remaining real zeros of $g(z)$

**E** Let $x^3 - 6x^2 + 11x - 6$ be a polynomial function with a known real zero of $c = 1$

i. Find the remaining real zeros of $g(z)$

**F*** Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with the property that $a_n + a_{n-1} + \cdots + a_1 + a_0 = 0$. (That is, the sum of the coefficents and the constant term is 0.)

i. Show that $(x - 1)$ is a factor of $f(x)$.

## 2.3

**A** Let $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$

i. Use Cauchy's Bound to find an interval containing all possible rational zeros.
ii. Use the Rational Zeros Theorem to make a list of possible rational zeros.
iii. Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros.

**B** Let $p(z) = 2z^4 + z^3 - 7z^2 - 3z + 3$

i. Use the Rational Zeros Theorem to list possible roots of the polynomial.
ii. Write the polynomial in factored form.

**C** Let $g(x) = x^4 - 9x^2 - 4x + 12$

i. Sketch the graph of $g(x)$.
ii. Label all axis intercepts on the graph.
iii. Write the end behavior of $g(x)$.

**D** Solve the following equation: $x^3 + x^2 = \dfrac{11x + 10}{3}$

**E** Let $f(x) = (x - 1)^2$ and $g(x) = 4$

i. Graph $f(x)$ and $g(x)$ on the same coordinate plane.
ii. Solve the inequality $f(x) \geq g(x)$ graphically.
iii. Solve the inequality $f(x) \geq g(x)$ algebraically and verify that it matches the solution found in part (ii).

**F** Solve the inequality: $\dfrac{x^3 + 20x}{8} \geq x^2 + 2$, express your answer in interval notation.

## 3.1

**A** Let $p(x) = 9x^3 + 5$ and $q(x) = 2x - 3$

i. Divide $p(x) \div q(x)$ using synthetic division or long division.

ii. Write $p(x)$ in the form of $d(x)q(x) + r(x)$.

**B** Let $p(x) = 4x^2 - x - 23$ and $q(x) = x^2 - 1$

i. Divide $p(x) \div q(x)$ using synthetic division or long division.

ii. Write $p(x)$ in the form of $d(x)q(x) + r(x)$.

**C** Let $h(x) = \dfrac{2x}{x+1}$.

i. Write $h(x)$ in the form $\dfrac{a}{x-h} + k$.

ii. Write the parent function $P(x)$ of $h(x)$.

iii. Track at least two points and the asymptotes from $P(x)$ and use them to graph $h(x)$.

**D** Let $r(x) = \dfrac{x^2 - x - 12}{x^2 + x - 6}$

i. Identify any holes in the graph of $r(x)$.

ii. Identify any vertical asymptotes in the graph of $r(x)$.

iii. State the domain of $r(x)$.

**E** Let $f(x) = \dfrac{x^3 + 2x^2 + x}{x^2 - x - 2}$

i. Identify any holes in the graph of $f(x)$.

ii. Identify any vertical asymptotes in the graph of $f(x)$.

iii. State the domain of $f(x)$.

**F*** Let $u(x)$ be a function defined only on the positive real numbers. Let $v(x) = (x - a)(x + b)$ with $0 < a < b$.

i. State the domain of $w(x) = \dfrac{u(x)}{v(x)}$

## 3.2

**A** Let $f(x) = 5x(6 - 2x)^{-1}$

i. Sketch the graph of $f(x)$. Label all asymptotes, holes, and zeros.

**B** Let $a(x) = \dfrac{x}{x^2 + x - 12}$

i. Sketch the graph of $a(x)$. Label all asymptotes, holes, and zeros.

**C** Let $r(t) = \dfrac{t^2 - t - 6}{t + 1}$

i. Sketch the graph of $r(t)$. Label all asymptotes, holes, and zeros.

**D** Let $f(x) = \dfrac{5x}{9 - x^2} - x$

i. Sketch the graph of $f(x)$. Label all asymptotes, holes, and zeros.

**E** Let $r(z) = -z - 2 + \dfrac{6}{3 - z}$

i. Sketch the graph of $r(z)$. Label all asymptotes, holes, and zeros.

**F*** Let $p(x) = 2x^3 + 5x^2 + 4x + 3$ and $q(x) = 2x + 1$

i. Does $r(x) = \dfrac{p(x)}{q(x)}$ have a horizontal or slant asymptote?

ii. Divide $p(x) \div q(x)$ and ignore the remainder. What does this suggest about the (non vertical) asymptote of $r(x)$?

iii. Assume $a(x)$ is a fourth degree polynomial, and $b(x)$ is a linear. Assuming $b(x)$ is not a factor of $a(x)$, what might the (non vertical) asymptote of $f(x) = \dfrac{a(x)}{b(x)}$ look like?

## 3.3

**A** Solve $\dfrac{3x-1}{x^2+1} = 1$.

**B** Solve $\dfrac{1}{t+3} + \dfrac{1}{t-3} = \dfrac{t^2-3}{t^2-9}$.

**C** Solve $\dfrac{4t}{t^2+4} \geq 0$.

**D** Solve $\dfrac{2t+6}{t^2+t-6} < 1$.

**E** Solve $\dfrac{3z-1}{z^2+1} \leq 1$.

**F*** Solve $\dfrac{2x^2-5x+4}{3x^2+1} < 0$, justify your answer.

## 4.1

**A** Let $f(x) = \sqrt{4-x} - 1$

i. Write the parent function $P(x)$ for $f$.

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

**B** Let $f(x) = -\sqrt[3]{8x+8} + 4$

i. Write the parent function $P(x)$ for $f$.

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

**C** Let $f(x) = -3\sqrt[4]{x-7} + 1$

i. Write the parent function $P(x)$ for $f$.

ii. Track at least three points from $P(x)$ and use them to graph $f(x)$.

**D** Let $d(x) = \dfrac{5x}{\sqrt[3]{x^3+8}}$

i. State the domain of $d(x)$.

**E** Let $z(x) = \sqrt{x(x+5)(x-4)}$

i. State the domain of $z(x)$.

**F** Let $c(x) = \sqrt[6]{\dfrac{x^2+x-6}{x^2-2x-15}}$

i. State the domain of $c(x)$.

## 4.2

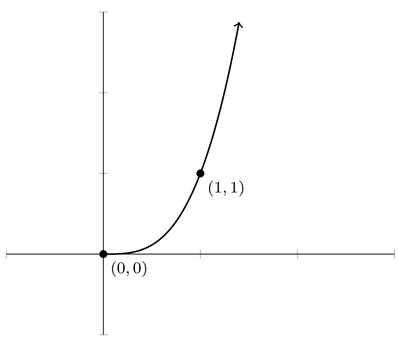
**A** Let $c(x) = x^{\frac{4}{7}}$

i. List the intervals where $c(x)$ is increasing (if any exist).
ii. List the intervals where $c(x)$ is decreasing (if any exist).
iii. List the intervals where $c(x)$ is concave up (if any exist).
iv. List the intervals where $c(x)$ is concave down (if any exist).

**B** Let $b(t) = t^{\frac{10}{4}}$

i. List the intervals where $c(x)$ is increasing (if any exist).
ii. List the intervals where $c(x)$ is decreasing (if any exist).
iii. List the intervals where $c(x)$ is concave up (if any exist).
iv. List the intervals where $c(x)$ is concave down (if any exist).

**C** The graph $g(t) = t^{\pi}$ is shown (where $\pi \approx 3.1415\ldots$).




i. Track the points provided on $g(t)$ to graph $G(t) = \left(\frac{t+3}{2}\right)^{\pi} - 1$

**D** Let $f(x) = x^{\frac{3}{2}}(x-7)^{\frac{1}{3}}$

i. State the domain of $f(x)$.

**E** Let $g(t) = t^{\frac{3}{2}}(t-2)^{-\frac{1}{2}}$

i. State the domain of $f(x)$.

**F*** Let $g(t) = 4t(9-t^2)^{-\sqrt{2}}$

i. State the domain of $g(t)$.

## 4.3

**A** Solve the equation $2x + 1 = (3 - 3x)^{\frac{1}{2}}$

**B** Solve the equation $(2x + 1)^{\frac{1}{2}} = 3 + (4 - x)^{\frac{1}{2}}$

**C** Solve the equation $2t^{\frac{1}{3}} = 1 - 3t^{\frac{2}{3}}$

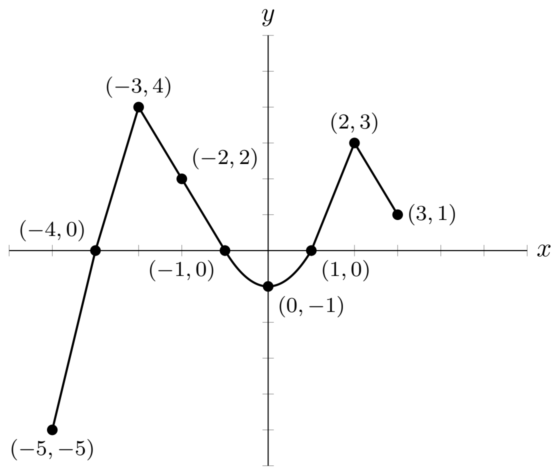
**D** Solve the inequality $\sqrt[3]{x} > x$, express your answer in interval notation.

**E** Solve the inequality $(2 - 3x)^{\frac{1}{3}} > 3x$, express your answer in interval notation.

**F** Solve the inequality $3(x - 1)^{\frac{1}{3}} + x(x - 1)^{-\frac{2}{3}} \geq 0$, express your answer in interval notation.

## 5.1

**A** Given the graph provided, answer all of the following questions.




$y = f(x)$

(a) Find the domain of $f$

(b) Find the range of $f$

(c) Find the maximum, if it exists.

(d) Find the minimum, if it exists.

(e) List the local maximums, if any exist.

(f) List the local minimums, if any exist.

(g) List the intervals where $f$ is increasing.

(h) List the intervals where $f$ is decreasing.

(i) Determine $f(-2)$.

(j) Solve $f(x) = 4$.

(k) List the $x$-intercepts, if any exist.

(l) List the $y$-intercepts, if any exist.

(m) Find the zeros of $f$.

(n) Solve $f(x) \geq 0$.

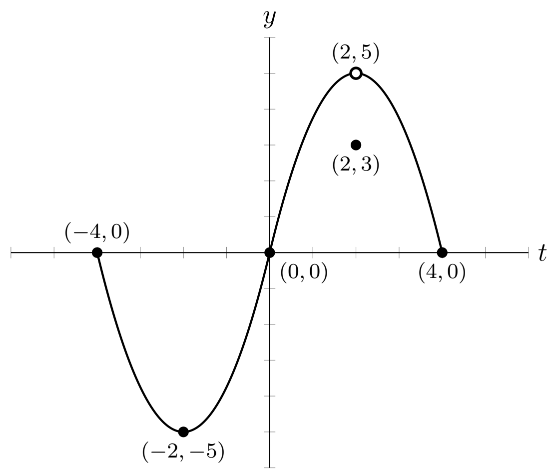
(o) Find the number of solutions to $f(x) = 1$.

(p) Find the number of solutions to $|f(x)| = 1$.

(q) Solve $(x^2 - x - 2)f(x) = 0$.

(r) Solve $(x^2 - x - 2)f(x) > 0$.

**B** Given the graph provided, answer all of the following questions.




$y = g(t)$

(a) Find the domain of $g$.

(b) Find the range of $g$.

(c) Find the maximum, if it exists.

(d) Find the minimum, if it exists.

(e) List of the local maximums, if any exist.

(f) List the local minimums, if any exist.

(g) List the intervals where $g$ is increasing.

(h) List the intervals where $g$ is decreasing.

(i) Determine $g(2)$.

(j) Solve $g(t) = -5$

(k) List the $t$-intercepts, if any exists.

(l) List the $y$-intercepts, if any exist.

(m) Find the zeros of $g$.

(n) Solve $g(t) \leq 0$.

(o) Find the domain of $G(t) = \dfrac{g(t)}{t+2}$

(p) Solve $\dfrac{g(t)}{t+2} \leq 0$

(q) How many solutions are there to $[g(t)]^2 = 9$?

(r) Does $g$ appear to be even, odd, or neither?

## 5.2

**A** Let $f(x) = 2x$ and $g(t) = \dfrac{1}{2t+1}$. Compute the indicated value if it exists.

i. $(f+g)(2)$

ii. $\left(\frac{f}{g}\right)(0)$

iii. $(fg)\left(\frac{1}{2}\right)$

**B** Let $f$ be the function defined by

$$f = \{(-3,4),(-2,2),(-1,0),(0,1),(1,3),(2,4),(3,-1)\}$$

and let $g$ be the function defined by

$$g = \{(-3,-2),(-2,0),(-1,-4),(0,0),(1,-3),(2,1),(3,2)\}$$

Compute the indicated value if it exists.

i. $(g+f)(1)$

ii. $\left(\frac{f}{g}\right)(-2)$

iii. $(gf)(-3)$

**C** Let $f(x) = x-1$ and $g(x) = \dfrac{1}{x-1}$, simplify the following expressions.

i. $(f+g)(x)$

ii. $(f-g)(x)$

iii. $(fg)(x)$

iv. $\left(\frac{f}{g}\right)(x)$

**D** Let $r(x) = \dfrac{3-x}{x+1}$.

i. Find nontrivial[1] functions $f$ and $g$ so that $r = fg$.

**E** Let $f(x) = -3x+5$.

i. Find and simplify the difference quotient using the formula: $\frac{f(x+h)-f(x)}{h}$

**F** Let $f(x) = x - x^2$.

i. Find and simplify the difference quotient using the formula: $\frac{f(x+h)-f(x)}{h}$

[1] Functions like $f(x) = 1$ do not count.

## 5.3

**A** Let $f(x) = 4x + 5$ and $g(t) = \sqrt{t}$, compute the following compositions, if any exist.

i. $(g \circ f)(0)$
ii. $(f \circ f)(2)$
iii. $(g \circ f)(-3)$

**B** Let $f$ be the function defined by

$$f = \{(-3,4), (-2,2), (-1,0), (0,1), (1,3), (2,4), (3,-1)\}$$

and let $g$ be the function defined by

$$g = \{(-3,-2), (-2,0), (-1,-4), (0,0), (1,-3), (2,1), (3,2)\}$$

Compute the indicated value if it exists.

i. $(f \circ g)(3)$
ii. $(f \circ g)(-3)$
iii. $g(f(g(0)))$
iv. $f(f(f(f(f(1)))))$

**C** Let $f(x) = x^2 - x + 1$ and $g(t) = 3t - 5$. Simplify the indicated composition.

i. $(g \circ f)(x)$
ii. $(f \circ g)(t)$

**D** Let $f(x) = x^2 - x - 1$ and $g(t) = \sqrt{t - 5}$. Simplify the indicated composition.

i. $(g \circ f)(x)$
ii. $(f \circ g)(t)$

**E** Let $f(x) = -2x$, $g(t) = \sqrt{t}$, and $h(s) = |s|$. Simplify the indicated composition.

i. $(f \circ g \circ h)(s)$
ii. $(h \circ f \circ g)(t)$
iii. $(g \circ h \circ f)(x)$

**F** Write $c(x) = \dfrac{x^2}{x^4 + 1}$ as a composition of two or more non-identity functions.

## 5.4

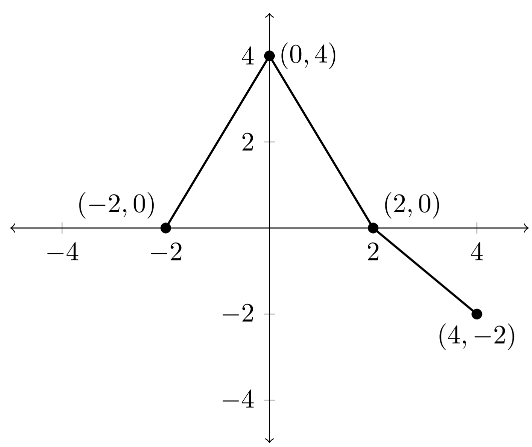
**A** Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $y = 3f(2x) - 1$.

**B** Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $y = 5f(2x + 1) + 3$.

**C** Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $f\left(\dfrac{7 - 2x}{4}\right)$.

**D** Suppose $(2, -3)$ is on the graph of $y = f(x)$. Using function transformations, find a point on the graph of $\dfrac{4 - f(3x - 1)}{7}$.

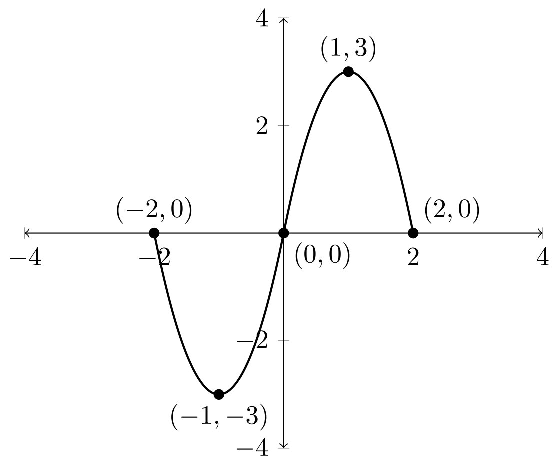
**E** Given the graph $y = g(t)$




$y = g(t)$

i. Graph the transformation $\frac{1}{2}g(t + 1) - 1$

**F** Given the graph $y = S(t)$




$y = S(t)$

i. Graph the transformation $y = \frac{1}{2}S(-t+1)+1$

## 5.5

**A** Graph the indicated relation in the $xy$-plane.

i. $\left\{(n, 4-n^2) \mid n = 0, \pm 1, \pm 2\right\}$

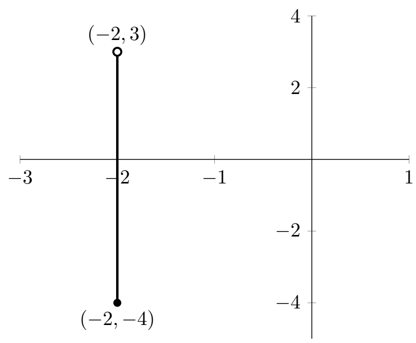
**B** Graph the indicated relation in the $xy$-plane.

i. $\{(3, y) \mid -4 \leq y < 3\}$

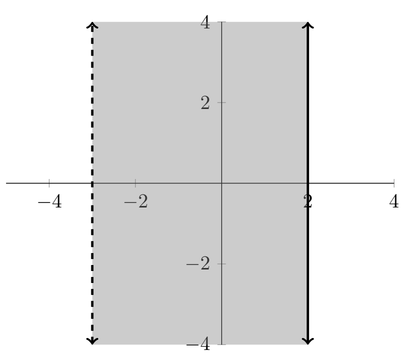
**C** Graph the indicated relation in the $xy$-plane.

i. $\{(x, y) \mid x \leq 3, y < 2\}$

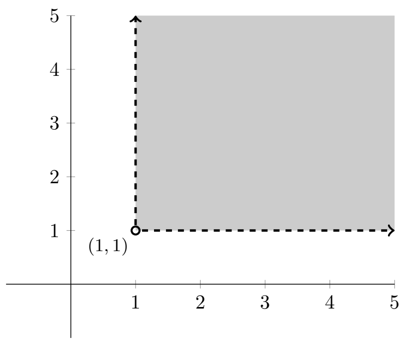
**D** Describe the given relation using set-builder notation.




**E** Describe the given relation using set-builder notation.



**F** Describe the given relation using set-builder notation.



## 5.6

**A** Let $f(x) = 2x + 7$ and $g(x) = \dfrac{x-7}{2}$.

i. Graph $f(x)$ and $g(x)$ on a coordinate plane.

ii. Are $f(x)$ and $g(x)$ inverse? Justify your answer.

**B** Let $g(t) = \dfrac{t-2}{3} + 4$.

i. Show that $g(t)$ is one-to-one.

ii. Find the inverse of $g(t)$.

**C** Let $f(x) = \sqrt{3x-1} + 5$.

i. Show that $f(x)$ is one-to-one.

ii. Find the inverse of $f(t)$.

**D** Let $f(x) = \sqrt[5]{3x-1}$

i. Show that $f(x)$ is one-to-one.

ii. Find $f^{-1}(x)$.

**E** Let $h(x) = \dfrac{2x-1}{3x+4}$

i. Show that $h(x)$ is one-to-one

ii. Find $h^{-1}(x)$.

**F*** Under what conditions is $f(x) = mx + b$, $m \neq 0$ its own inverse? Prove your answer.

## 6.1

**A** Let $f(x) = 3^x$.

i. Sketch the graph of $f(x)$.

ii. Using transformations, graph $g(x) = 3^{-x} + 2$.

**B** Let $f(x) = 10^x$

i. Sketch the graph of $f(x)$.

ii. Using transformations, graph $g(x) = 10^{\frac{x+1}{2}} - 20$.

**C** Let $f(t) = e^t$

i. Sketch the graph of $f(t)$.

ii. Using transformations, graph $g(t) = 8 - e^{-t}$.

**D** State the domain of $T(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

## 6.2

**A** Rewrite the expression: $\log(100) = 2$, so that it does not contain a logarithm.

**B** Evaluate $\log_2(32)$.

**C** Evaluate $\log_4(8)$.

**D** Find the domain of $f(x) = \log_7(t^2 + 9t + 18)$.

**E** Find the domain of $f(x) = \ln(x^2 + 1)$.

**F** Find the domain of $g(t) = \ln(7 - t) + \ln(t - 4)$.

## 6.3

**A** Expand and simplify: $\ln\left(\dfrac{\sqrt{z}}{xy}\right)$.

**B** Expand and simplify: $\ln\left(\sqrt[4]{\dfrac{xy}{ez}}\right)$.

**C** Write $\frac{1}{2}\log_3(x) - 2\log_3(y) - \log_3(z)$ as a single logarithm.

**D** Write $\log_5(x) - 3$ as a single logarithm.

**E** Write $\log_2(x) + \log_4(x)$ as a single logarithm.

**F*** With the product rule given, prove the quotient rule and power rule for logarithms.

## 6.4

**A** Solve $2^{(t^3-t)} = 1$.

**B** Solve $3^{7x} = 81^{4-2x}$.

**C** Solve $e^{2t} = e^t + 6$.

**D*** Solve $7^{3+7x} = 3^{4-2x}$.

**E** Solve $e^{-x} - xe^{-x} \geq 0$, write your answer in interval notation.

**F** Solve $(1 - e^t)t^{-1} \leq 0$, write your answer in interval notation.

## 6.5

**A** Solve $10\log\left(\frac{x}{10^{-12}}\right) = 150$.

**B** Solve $3\ln(t) - 2 = 1 - \ln(t)$.

**C** Solve $\ln(x+1) - \ln(x) = 3$.

**D** Solve $\ln(t^2) = (\ln(t))^2$.

**E** Solve $\frac{1-\ln(t)}{t^2} < 0$, write your answer in interval notation.

**F*** Solve $\ln(t^2) \leq (\ln(t))^2$, write your answer in interval notation.

## 7.1

**A** Convert $135°$ into radians.

**B** Convert $\frac{5\pi}{3}$ into degrees.

**C** Let $\theta = \frac{15\pi}{4}$

i. Graph $\theta$ in standard position.

ii. Give two angles coterminal to $\theta$, one which is positive and one which is negative.

**D** Let $\theta = -\frac{13\pi}{6}$

i. Graph $\theta$ in standard position.

ii. Give two angles coterminal to $\theta$, one which is positive and one which is negative.

## 7.2

**A** Given $\theta = \dfrac{3\pi}{4}$

i. Find the value of $\sin(\theta)$.

ii. Find the value of $\cos(\theta)$.

**B** Find all angles which satisfy the equation: $\sin(\theta) = \dfrac{\sqrt{3}}{2}$

**C** Let $\theta$ be an angle in standard position whose terminal side contains the point $P(5, -9)$.

i. Compute $\cos(\theta)$.

ii. Compute $\sin(\theta)$.

**D** Assume $\cos(\theta) = -\dfrac{2}{11}$ with $\theta$ in Quadrant III.

i. Find the value of $\sin(\theta)$.

**E** Assume $\sin(\theta) = \dfrac{2\sqrt{5}}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$.

i. Find the value of $\cos(\theta)$.

**F** Draw the unit circle from memory.[2]

[2] This would not be asked on a test, but you should be able to do this.

## 7.3

**A** Let $f(t) = \cos\left(t - \frac{\pi}{2}\right)$

i. State the amplitude, baseline, period, and phase shift of $f(t)$.

ii. Graph one cycle of $f(t)$.

**B** Let $g(t) = \sin(2t - \pi)$

i. State the amplitude, baseline, period, and phase shift of $g(t)$.

ii. Graph one cycle of $g(t)$.

**C** Let $h(t) = \cos(3t - 2\pi) + 4$

i. State the amplitude, baseline, period, and phase shift of $h(t)$.

ii. Graph one cycle of $h(t)$.

**D** Let $q(t) = -\frac{3}{2}\cos\left(2t + \frac{\pi}{3}\right) - \frac{1}{2}$

i. State the amplitude, baseline, period, and phase shift of $q(t)$.

ii. Graph one cycle of $q(t)$.

**E*** Let $S$ be a collection of sine functions

$$S = \{\sin(\omega_0 x),\, \sin(\omega_1 x),\, \sin(\omega_2 x), \ldots,\, \sin(\omega_n x)\}$$

where no two values of $\omega$ are the same. Find a value of $x$, other than $x = 0$, where all of the sine functions in $S$ equal 0 at the same time.

## 7.4

**A** Find the value of $\csc\left(\frac{5\pi}{6}\right)$ if it exists.

**B** Find the value of $\sec\left(-\frac{3\pi}{2}\right)$ if it exists.

**C** If it is known that $\sin(\theta) > 0$ but $\tan(\theta) < 0$, in what quadrant does $\theta$ lie?

**D** Assume $\tan(\theta) = \frac{12}{5}$ with $\theta$ in Quadrant III.

i. Find the value of the other five circular functions.

**E** Assume $\cot(\theta) = 2$ with $0 < \theta < \frac{\pi}{2}$

i. Find the value of the other five circular functions.

**F** Find all angles which satisfy the equation $\tan(\theta) = -1$

## 7.5

**A** Let $f(t) = \sec(3t - 2\pi) + 4$

i. State the period of $f(t)$.

ii. Graph one cycle of $f(t)$.

**B** Let $g(t) = \csc\left(-t - \frac{\pi}{4}\right) - 2$

i. State the period of $g(t)$.

ii. Graph one cycle of $g(t)$.

**C** Let $r(t) = 2\tan\left(\frac{1}{4}t\right) - 3$

i. State the period of $r(t)$.

ii. Graph one cycle of $r(t)$.

**D** Let $s(t) = \cot\left(t + \frac{\pi}{6}\right)$

i. State the period of $s(t)$.

ii. Graph one cycle of $s(t)$.

## 8.1

**A** Verify the identity: $\dfrac{\cos(\theta)}{\sin^2(\theta)} = \csc(\theta)\cot(\theta)$

**B** Verify the identity: $\dfrac{\cos(t)}{1-\sin^2(t)} = \sec(t)$

**C** Verify the identity: $\tan^3(t) = \tan(t)\sec^2(t) - \tan(t)$

**D** Verify the identity: $\dfrac{1-\tan(t)}{1+\tan(t)} = \dfrac{\cos(t)-\sin(t)}{\cos(t)+\sin(t)}$

**E** Verify the identity: $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$

**F** Verify the identity: $\dfrac{1-\sin(x)}{1+\sin(x)} = (\sec(x)-\tan(x))^2$

## 8.2

**A** Find the exact value of $\cos\left(\dfrac{13\pi}{12}\right)$

**B** Find the exact value of $\sin\left(\dfrac{\pi}{12}\right)$

**C** Let $\alpha$ be a Quadrant IV angle such that $\cos(\alpha) = \frac{\sqrt{5}}{5}$ and let $\frac{\pi}{2} < \beta < \pi$ such that $\sin(\beta) = \frac{\sqrt{10}}{10}$.

i. Find the value of $\cos(\alpha - \beta)$.

**D** Let $0 < \alpha < \frac{\pi}{2}$ such that $\csc(\alpha) = 3$ and let $\beta$ be a Quadrant II angle such that $\tan(\beta) = -7$.

i. Find the value of $\tan(\alpha + \beta)$.

**E** Verify the identity: $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos(\alpha)\cos(\beta)$.

**F** Verify the identity: $(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$.

## 8.3

**A** Find the exact value of $\arccos\left(\frac{1}{2}\right)$

**B** Find the exact value of $\operatorname{arccot}(-1)$

**C** Find the exact value of $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$

**D** Find the exact value of $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$

**E** Solve $\sin(\theta) = \frac{7}{11}$

**F** State the domain of $\arctan(4x)$

## 9.1

*Chapter 9 is often not included in a final exam.*

**A** Let $(\alpha, a)$, $(\beta, b)$, and $(\gamma, c)$ be angle-side opposite pairs of a triangle such that $\alpha = 13°$, $\beta = 17°$, and $a = 5$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.

**B** Let $(\alpha, a)$, $(\beta, b)$, and $(\gamma, c)$ be angle-side opposite pairs of a triangle such that $\alpha = 73.2°$, $\beta = 54.1°$, and $a = 117$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.

**C** Let $(\alpha, a)$, $(\beta, b)$, and $(\gamma, c)$ be angle-side opposite pairs of a triangle such that $\alpha = 95°$, $\beta = 85°$, and $a = 33.33$.

i. Does this information produce a triangle? If so, find the remaining values. If not, explain.

## 9.2

*Chapter 9 is often not included in a final exam.*

**A** Find the area of the triangle with side lengths, $a = 7$, $b = 10$, and $c = 13$.

**B** Find the area of the triangle with side lengths, $a = 300$, $b = 302$, and $c = 48$.[3]

**C** Find the area of the triangle with side lengths, $a = 5$, $b = 12$, and $c = 13$.

[3] Use a calculator.