The following checklist will help you sketch a curve by hand. Not every item is used on every curve, but it is a good idea to check each item before drawing a graph. We will assume our curve is a function $y = f(x)$.

A. Domain

- Common domain restrictions
  - Division by 0: (common with rational functions)
  - Even roots: examples are $\sqrt{x}$, $\sqrt[4]{x}$, $\sqrt[6]{x}$, e.t.c. Set the contents under the root ≥ 0 and solve to find domain restriction.
  - Logarithms: any logarithm has a restricted domain. To check, set the contents inside the logarithm > 0 and solve.

B. Intercepts

- y-intercepts
  - Set all instances of x to 0 and solve.
- x-intercepts (sometimes called roots or zeroes)
  - Set the equation equal to 0 and solve. This step may need to be skipped if the equation is difficult to solve.

C. Symmetry

- Even functions
  - Have the property: $f(-x) = f(x)$. If your function is even, you only need to plot $x \geq 0$, then you can reflect over the y-axis.
- Odd functions
  - Have the property: $f(-x) = -f(x)$. If your function is odd, you only need to plot $x \geq 0$, then rotate 180° about the origin.
- Periodic functions
  - Have the property: $f(x + p) = f(x)$ for some positive constant p (most common examples are trigonometric functions). If f is periodic, you only need to graph one period, and then you can replicate that period multiple times.

D. Asymptotes

- Horiztonal Asymptotes occur in the following cases:
  - $\lim_{x\to\infty} f(x) = L$
  - $\lim_{x\to-\infty} f(x) = L$

  in which the line $y = L$ is a horiztonal asymptote.
- Vertical Asymptotes occur in the following cases:
  - $\lim_{x\to a^+} f(x) = \infty$
  - $\lim_{x\to a^+} f(x) = -\infty$
  - $\lim_{x\to a^-} f(x) = \infty$
  - $\lim_{x\to a^-} f(x) = -\infty$

  in which $x = a$ is a vertical asymptote.

- Slant Asymptotes occur when
  - $\lim_{x\to\infty}[f(x) - (mx + b)] = 0$

  and $m \neq 0$. These are most often found on rational functions where the degree of the numerator is one larger[1] than that of the denominator, and long division is an easy way of finding the equation of the slant asymptote.

E. Intervals of Increase or Decrease

- Compute $f'(x)$ and find the intervals on which $f'(x)$ is positive / negative. These intervals correspond to $f(x)$ increasing / decreasing.

F. Local Maximum or Minimum Values

- Critical numbers occur when $f'(x)$ is 0 or undefined.
- If $f'(x)$ changes from positive to negative as it passes a critical number, $f(x)$ has a local maximum at that point.
- If $f'(x)$ changes from negative to positive as it passes a critical number, $f(x)$ has a local minimum at that point.

G. Concavity and Points of Inflection

- Compute $f''(x)$. Find the intervals on which $f''(x)$ is positive / negative. These intervals correspond to $f(x)$ being concave up / concave down.
- Inflection points occur when $f''(x)$ changes from negative to positive or vice versa.

H. Sketch the Curve

- Sketch asymptotes as dashed lines.
- Plot the intercepts, maximum and minimum values, and inflection points.
- Make your curve pass through these points, but make sure it rises and falls according to **E**, and has concavity according to **G**, and approaches asymptotes when they occur.
- If you need additional accuracy at any point, you can compute the derivative at that point. The value of the derivative will indicate the direction in which the function is moving at that point.

[1] You can actually have a "slant" asymptote that is a higher degree than that of a linear function. A rational function where the degree of numerator is n larger than the denominator, would have a degree n function as its asymptote. Again, long division works for finding these.